Comments on the Channel Capacity of Ethernet

Maher H. Heal
College of computer Science and
Information Technology,
Abu Dhabi University, Al-Ain, UAE

Abstract-Ethernet is the most widely used network and many analytical models were developed to predict its capacity. Usually in these models assumptions like infinite population, no back-off algorithm and slotted channel are made. By comparing two models (one slotted and the other non-slotted) with reported measurements, we showed the optimistic estimate of the channel capacity of the slotted model is not due to slotting the channel as believed in the literature but due to other assumptions. Finally, the effect of difference between IEEE 802.3 and Ethernet DIX version 1 regarding preamble sending (IEEE 802.3 completes sending the preamble and then sends the jam signal while Ethernet DIX version 1 aborts sending the preamble and sends the jam signal immediately) on degrading the channel capacity was analyzed.

Keywords: Ethernet, IEEE 802.3, CSMA/CD, preamble, interframe gap.

I. Introduction

Ethernet is the most commonly used local area network and being such a network many analytical models have been introduced in the literature to analyze its performance [1, 2, 10]. It is usually difficult to analyze the interaction of all the protocols that constitute Ethernet and to obtain their effect on its performance, so many analyses focused mainly on the most effective protocol on Ethernet performance which is CSMA/CD protocol. But even for that protocol, many simplifying assumptions are made to make the mathematical model tractable.

In this paper we compare two models: F. A. Tobagi et. al. [1] and K. Sohraby et. al. [2] which we call SMV with reported measurements. The first model has been used widely in the literature to predict the performance of Ethernet [3, 4, 5]. We discuss the effect of making simplifying assumptions and we show that the optimistic estimates of the channel capacity of the first model is not due to the slotting assumption as it is believed in the literature [3]. Then, we modify the SMV model to study the effect of sending the preamble and hence we have a more realistic model. Indeed the issues of interframe gap and preamble have been largely ignored in traditional CSM/CD models while we discuss them here.

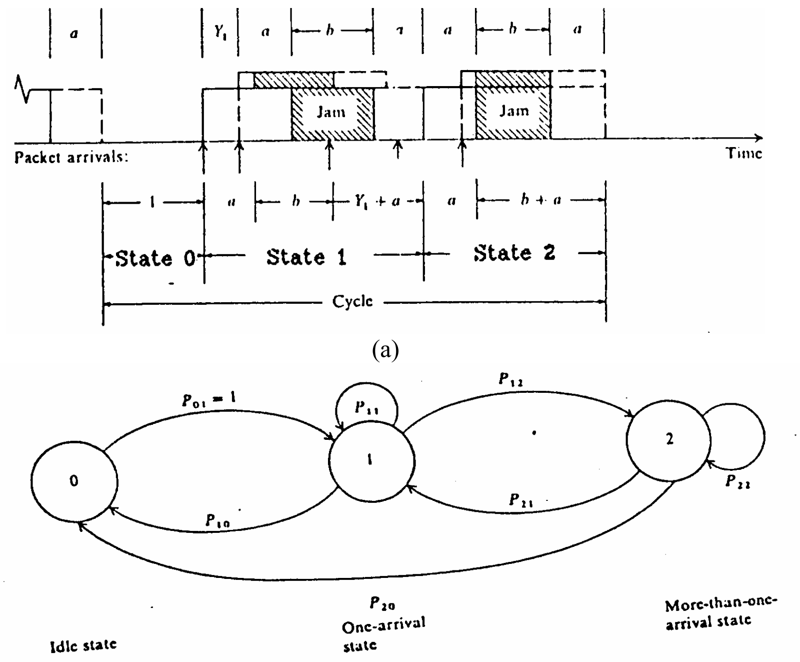
II. Comparison of SMV and Tobagi models with measurements

Here we present a summary of the SMV [2] and Tobagi [1] models and the assumptions used by each model and how they differ from Ethernet; SMV model is described in more details as we will use that model in section III to cover the preamble issue. For complete details of the analyses, the reader is referred to the papers cited above.

Soharby et. al.[2] developed a model of 1-persistent CSMA/CD by assuming an infinite population model where the total packet generation (old and new) follows a Poisson distribution with an infinitesimally generation rate per station; thus every collided packet has to wait for infinite time before attempting sending it again and thus no backoff algorithm was used (Ethernet uses truncated binary exponential backoff algorithm). The topology used was worst case star topology (Ethernet DIX version 1 uses bus topology) and the interfame gap is zero, so that a station may transmit the packet immediately as soon as sensing carrier-off signal (Ethernet waits for an interframe gap and then sends the packet).

By using the above assumptions a model of three Markov states which are listed below was developed, see Fig. (1):

(1) State 0: This is the idle state which the system begins in. The system enters this state when transmission ends and there is no deferring station (state 1→state 0, state 2→state 0).
(2) State 1: The system enters this state when there is one and only one station starts transmission. This can be done by a station initiating transmission of a newly generated packet while the system is in state 0 (state 0→state 1) or by a deferring station (state 1→state 1, state 2→state 1).
(3) State 2: The system enters this state when the transmission is initiated by more than one deferring station (state 1→state 2, state 2→state 2).



(a)

(b)

Fig. 1. The SMV model. (a) Channel states. (b) Markov chain representation.

Thus, the throughput is given by:

$$S = \frac{\pi_1 e^{-aG}}{\sum_{i=0}^{2} E(T_i)\pi_i} \qquad (1)$$

The Appendix gives the equations used to calculate different terms in the above equation. Where:

a	end-to-end propagation delay normalized to the packet length.
b	length of jam signal normalized to the packet length.
G	packets (old and new) generation rate per unit time which is equal to packet length.
$E(T_i)$	average time the protocol spends in state i; i = 0, 1, 2.
π_i	stationary probability of state i; i = 0, 1, 2.
p_{ij}	transition probability from state i to state j. i, j = 0, 1, 2.

The SMV model can be modified very simply to incorporate the effect of the interframe gap. The transition probabilities and states average times of this modified model (SMV with gap) are given in the Appendix.

Tobagi used the same assumptions for SMV model but with a slotted channel; where the slot size is the end-to-end propagation delay. Both the SMV model (with gap and without gap) and Tobagi models were compared with reported measurements. The comparison is based on the channel capacity, the calculated one and the measured one. The result of the comparison is shown in table (1) and table (2). The measured and Tobagi's model values were taken from Gonsalve's paper [3], while those of SMV and SMV with gap models were obtained by drawing throughput (S) as a function of the load (G) curve and taking the maximum of that curve. Fig. (2) shows one such curve where the effect of interfarme gap on throughput and the channel capacity is clear. Throughputs calculated are net, excluding 4 bytes overhead and 4 bytes and 12 bytes values were assumed for the jam and interframe gap respectively. From the two tables, it is clear the channel capacity as predicted by SMV model is larger than that by Tobagi's model. This can be explained by noting that the collision duration in Tobagi's model is of constant length which is taken to be equal to the worst case (three times the end-to-end propagation delay + jam signal) while SMV model accounts for two collision durations; duration of state 2 which is equal to $2a + b$ and duration of state 1 which is less than the worst case in case of collision. Thus, for large (a) where the throughput is less sensitive to the collision duration length (this will be explained further in the next section), there is a little difference between the two models, but as (a) gets smaller the difference increases and finally as (a) gets very small the models give almost the same results since in that case the duration of state 1 (in case of collision) and state 2 of the SMV model are nearly equal to each other and to the worst case ($\approx$jam signal); in addition the effect of the optimistic assumption of slotting the channel is small. Thus, from the above discussion and since the SMV with gap model relaxes the slotted channel assumption and it is only different from Ethernet by the backoff algorithm and infinite population assumption[1] and still its predicted values are optimistic; one concludes that it is not the optimistic assumption of slotting the channel that gives rise to the higher values of the channel capacity as predicted by Tobagi's model, but the reason for that seems to be due to the infinite population assumption or due to the difference in the backoff algorithm[2]

Table (1) Comparison with measurement (750m + 1 repeater)

Packet length, bytes	a	Measured	Tobagi	SMV	SMV with gap
64	0.22	26	40	42	37
200	0.072	62	60	64	59
512	0.028	72	77	82	78
1500	0.0098	86	91	93	91
5000	0.0029	95	97	98	97

Table (2) Comparison with measurement (1500m + 2 repeaters)

Packet length, bytes	a	Measured	Tobagi	SMV	SMV with gap
64	0.28	26	37	38	33
200	0.092	60	58	60	56
512	0.036	72	74	78	75
1500	0.012	85	89	92	90
5000	0.0037	94	96	97	97
10000	0.0019	97	98	99	98

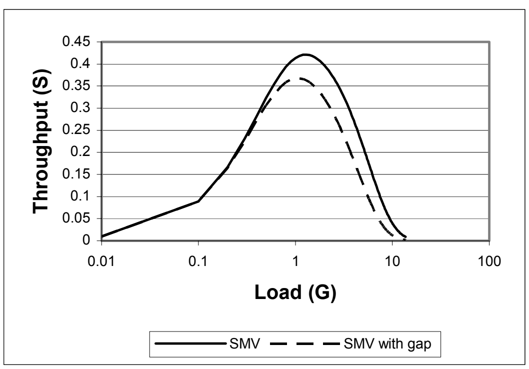


Fig. 2. Throughput as a function of load for SMV and SMV with gaps models, packet length= 64 and a =0.22.

[1] Although the model is different from Ethernet also by the assumed star topology, but that assumption is pessimistic while the model is giving higher values than the measured ones.

[2] Chikara Yasuda et. al. developed a model of a finite population 1-persistent CSMA/CD with a backoff algorithm that follows exponential distribution and they showed a maximum throughput can be achieved by selecting the appropriate mean of the exponential distribution of the backoff algorithm which depends on the number of stations.

III. IEEE 802.3 Channel Capacity

In Ethernet DIX version 1 standard [7], if a station while transmitting the preamble a collision detect signal is asserted, any remaining preamble bits shall not be sent by the station. The station should immediately proceed with the transmission of the jam signal. While in IEEE 802.3 [8] any remaining preamble bits should be sent and the jam signal is sent afterwards. This may be due an attempt to make the separation between the physical layer and the data link layer precise. In this section the effect of sending the preamble as described above on the channel capacity is investigated using the SMV model.

it is clear from the above that the collision period in IEEE 802.3 will be larger than that of Ethernet and hence a degradation in the performance is expected.

The following notation will be used in this section in addition to the notation used in section II:

- t_i — time of the ith transmission in the vulnerable period.
- e_i — time at which the ith transmitting station ceases transmission.
- e — time at which the last transmitting station in a collision period ceases transmission.
- p — preamble length, normalized to the packet length.
- n — number of transmitting stations in the vulnerable period.

In section II the last transmitting station in a collision period is the first transmitting one; but now, due to introducing the preamble this is not necessarily true. This is proved below.

The first transmitting station will detect collision at time $t_2 + a$ hence we have (see Fig. (3));

$$e_1 = t_2 + a + q(x_1) + b \qquad (2)$$

where $q(x_1)$ represents the remaining preamble bits, so:

$$q(x_1) = \begin{cases} p - x_1 & 0 \le x_1 \le p \\ 0 & x_1 \ge p \end{cases} \qquad (3)$$

and $$x_1 = t_2 - t_1 + a \qquad (4)$$

is what the first transmitting station has already transmitted when it detects collision.

On the other hand, all other transmitting stations will detect collision at time $t_1 + a$, hence $e_2 \le e_3 \le \ldots \le e_n$. The equality holds when all stations transmitted their preamble, when they detect collision; hence:

$$e = \max imum(e_1, e_n) \qquad (5)$$

$$e_n = t_1 + a + q(x_n) + b \qquad (6)$$

$$x_n = a - (t_n - t_1) \quad (7)$$

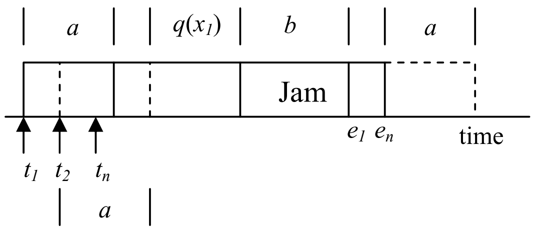


Fig. 3. Collision period.

Letting $z = a - p$; the z line will be divided into three regions (see Fig. (4)) according to the following inequalities:

$$q(x_1) = 0 \Leftrightarrow t_2 - t_1 + a \geq p \Leftrightarrow a - p \geq -(t_2 - t_1)$$
$$q(x_n) = 0 \Leftrightarrow a - (t_n - t_1) \geq p \Leftrightarrow a - p \geq t_n - t_1$$

From Fig. (4) and Eqs. (2), (3), (4), (6) and (7) we have now:

(1) $z \geq t_n - t_1 ; q(x_1) = q(x_n) = 0$

$e_1 = t_2 + a + b$

$e_n = t_1 + a + b$

(2) $-(t_2 - t_1) \leq z < t_n - t_1 ; q(x_1) = 0; q(x_n) \neq 0$

$e_1 = t_2 + a + b$

$e_n = t_n + p + b$

$thus, e_1 \geq e_n \Leftrightarrow t_2 - t_n \geq p - a \Leftrightarrow z \geq t_n - t_2$

(3) $z < -(t_2 - t_1); q(x_1) \neq 0; q(x_n) \neq 0$

$e_1 = t_1 + p + b$

$e_n = t_n + p + b$

From the above three cases and Eq.(5), we have:

$$\begin{aligned}&(1)\text{If } z \geq t_n - t_2 \text{ then } e = e_1 = t_2 + a + b\\ &(2)\text{If } z \leq t_n - t_2 \text{ then } e = e_n = t_n + p + b\end{aligned} \quad (8)$$

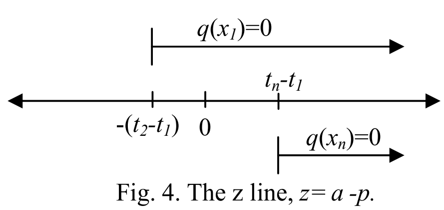


Fig. 4. The z line, $z = a - p$.

Two cases will be considered now:

I) a>p

By taking a wide range for (a) values from 0 to 0.5, and considering the two extreme cases of the original SMV model: (1) b= jam (normalized) and (2) b=jam + preamble (both normalized) and preamble= a [3] we found that the throughput vs. G curves are identical for small values of a and as a gets larger the difference between the two curves tails increases while the channel capacity remains nearly the same. This can be explained by approximating the throughput by:

$$S = \frac{1}{1 + mq}$$

$$\text{so, } \frac{\partial S}{\partial q} = -\frac{m}{(1 + mq)^2}$$

where m is the average number of collisions per successful transmission. and q is the average collision period length[4]. Now, when (a) is very small there are few collisions (m is small) but now there is also very small change in (q) (b=jam; b=jam + preamble and preamble= a) and hence S remains the same, also it is clear from $\partial S / \partial q$ the throughput is less sensitive to the change in collision period for small m. Now, let (a) large; (m) will be large and it is clear $\partial S / \partial q$ is small for large m independent of q. ($\partial S / \partial q \to 0$ as $m \to \infty$).

II) $a < p$

Since $z = a - p$ and so $z<0$; then from Eq. (8) $e = t_n + p + b$, so the length of the collision period will be

$$e - t_1 + a = Y_n + p + b + a \quad (9)$$

where $Y_n = t_n - t_1$ is a random variable has the probability distribution of

$$F_{Y_n}(y) = \Pr.(Y_n \leq y) = \begin{cases} 0 & y < 0 \\ e^{-G(a-y)} & 0 \leq y \leq a \\ 1 & y \geq a \end{cases} \quad (10)$$

Fig. (5) shows the duration of state 1 and state 2 of the Markov chain; Fig. (5-a) shows a successful transmission and Fig. (5-b) shows the duration of state 2 when the system enters that state by two or more deferring stations and no station is trying to initiate transmission in the vulnerable period a; while Fig. (5-c) shows state 1 or 2 when there is a station initiating transmission in the vulnerable period.

[3] It is clear the performance of IEEE 802.3 with preamble lies between these two extreme cases since we assume the largest collision period by taking b=jam +preamble and preamble= a .

[4] In Ethernet, when neglecting the preamble effect we have $2a + b \leq q \leq 3a + b$ and by considering the preamble effect in IEEE 802.3 we have $2a + p + b \leq q \leq 3a + p + b$; as $a > p$ the worst case can be found by having $p = a$.

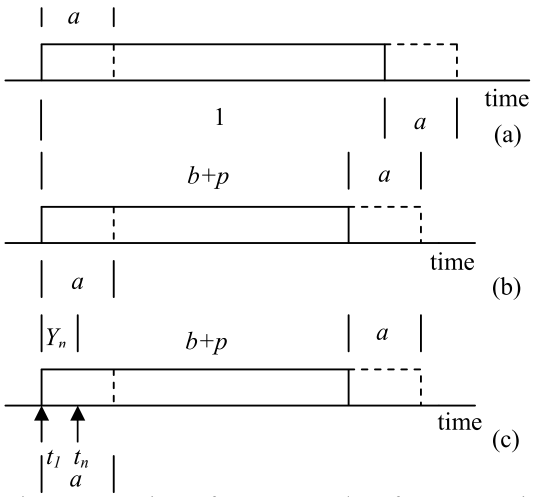


Fig. 5. Duration of state 1 and 2 for SMV with preamble model. (a) Duration of state 1 (T_1), successful transmission (b) Duration of state 2 (T_2), Y_n=0 (c) Duration of state 1 (T_1) or state 2 (T_2) , $Y_n \neq$ 0.

The transition probabilities and states average times are derived below using the new collision period in Eq. (9). The derivation completely parallels that in [2], hence it is described briefly.

(1) The transition probabilities (see Fig. (5))

$$p_{01} = 1 \tag{11}$$

$$\begin{aligned} p_{10} &= \Pr.\{\text{no arrival during 1, success}\} \\ &+ \Pr.\{\text{no arrival in } y+p+b, \text{ collision}\} \\ &= e^{-G(1+a)} + \int_0^a e^{-G(y+p+b)} G e^{-(a-y)} dy \\ &= e^{-G(1+a)} + Gae^{-G(a+p+b)} \end{aligned} \tag{12}$$

$$\begin{aligned} p_{11} &= \Pr.\{\text{one arrival in interval 1,success}\} \\ &+ \Pr.\{\text{one arrival in } y+p+b, \text{ collision}\} \\ &= \mathrm{G}e^{-\mathrm{G}(1+\mathrm{a})} + G^2 ae^{-G(a+p+b)}[\frac{1}{2}a+p+b] \end{aligned} \tag{13}$$

$$\begin{aligned} p_{20} &= \Pr.\{\text{zero arrivals in } p+b, Y_n = 0\} \\ &+ \Pr.\{\text{zero arrivals in } y+p+b, Y_n \neq 0\} \\ &= e^{-G(a+p+b)}(1+Ga) \end{aligned} \tag{14}$$

$$\begin{aligned} p_{21} &= \Pr.\{\text{one arrival in } p+b, Y_n = 0\} \\ &+ \Pr.\{\text{one arrival in } y+p+b, Y_n \neq 0\} \\ &= G(p+b)e^{-G(a+p+b)} + G^2 ae^{-G(a+p+b)}[\frac{1}{2}a+p+b] \end{aligned} \tag{15}$$

(2) States average times (see Fig. (5))

$$E(T_0) = 1/G \tag{16}$$

$$\begin{aligned} E(T_1) &= E\{T_1 / Successful\}.\Pr.\{successful\} \\ &+ E\{T_1 / collision\}\Pr.\{collision\} \\ &= (1+a)e^{-Ga} + E(Y_n+p+b+a / collision)(1-e^{-Ga}) \\ &= e^{-Ga} + 2a + (1-e^{-Ga})(p+b-\frac{1}{G}) \end{aligned} \tag{17}$$

$$\begin{aligned} E(T_2) &= E(T_2 / Y_n = 0)\Pr.(Y_n = 0) + E(T_2 / Y_n \neq 0)\Pr.(Y_n \neq 0) \\ &= (p+b+a)e^{-Ga} + E(Y_n+p+b+a / Y_n \neq 0)\Pr.(Y_n \neq 0) \\ &= 2a+b+p-\frac{1}{G}(1-e^{-Ga}) \end{aligned} \tag{18}$$

IV. Results

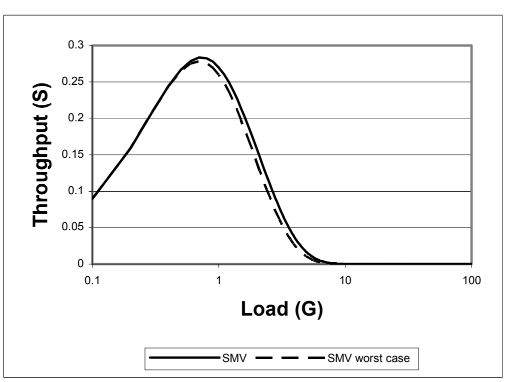
In Figs. (6) and (7) the throughput (S) as a function of load was drawn for different values of (a) where the following practical values were used:

Total Packet length (without preamble)=64 bytes. Jam signal= 4bytes.

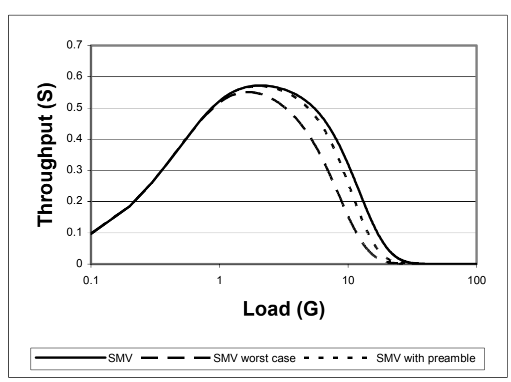
(1)preamble = 8 bytes in Fig. (6).

(2)preamble = 16 bytes in Fig. (7) [9]

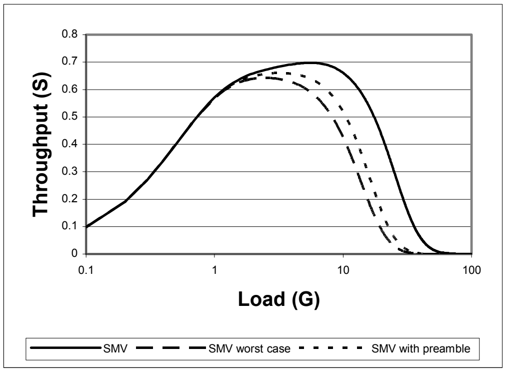
It is clear from both figures that the difference between SMV model (without preamble, i.e., b=jam) and SMV worst case (b=jam + preamble) gets more significant as (a) gets smaller and the IEEE 802.3 performance as derived in section (III) lies between the two, and it is nearer to the former for large a and to the later for small a completely as expected and as explained in the case I $a > p$.

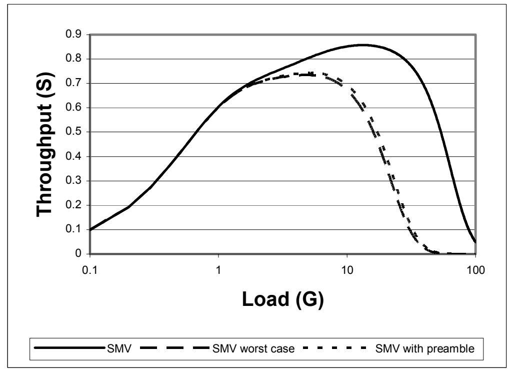


(a)



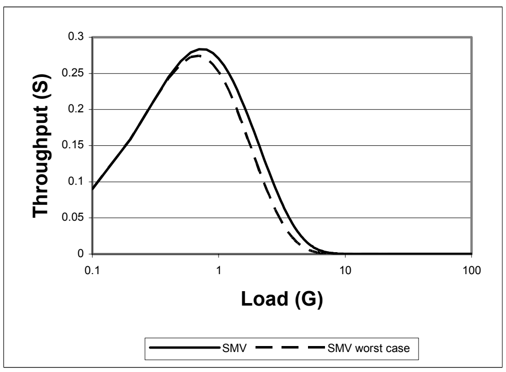
(b)

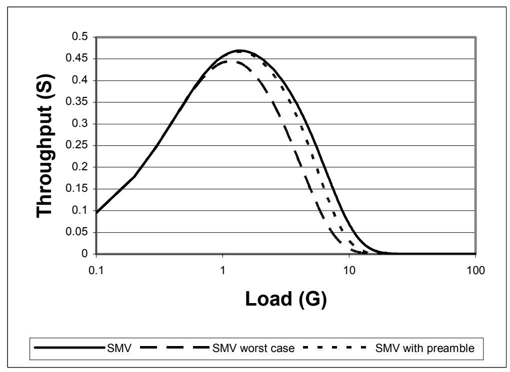




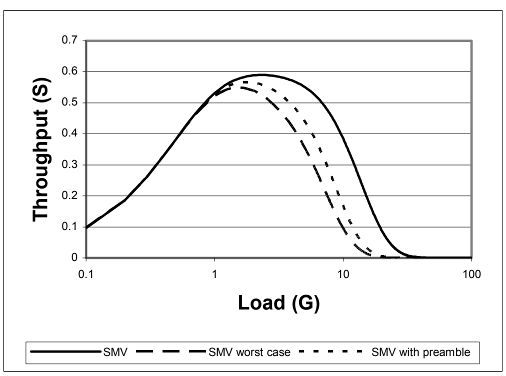
(c) (d)

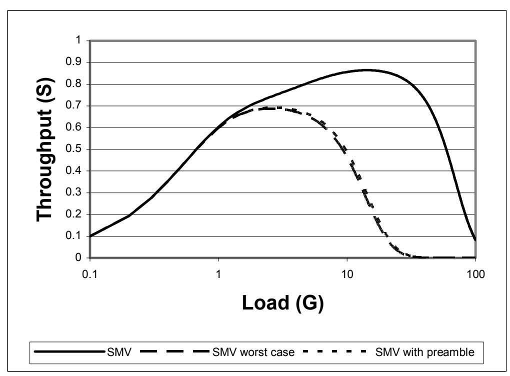
Fig. 6. IEEE 802.3 parameters. (a) $a > p$, $a = 0.5$ (b) $a = p$, $a = 0.111$ (c) $a < p$, $a = 0.05$ (d) $a < p$, $a = 0.01$.





(a) (b)





(c) (d)

Fig. 7. 82586 parameters. (a) $a > p$, $a = 0.5$ (b) $a = p$, $a = 0.2$ (c) $a < p$, $a = 0.1$ (d) $a < p$, $a = 0.01$.

IV. Conclusion

Two widely used models in the literature to calculate the channel capacity of Ethernet were compared with reported measurements. One of the models is slotted and the other is non-slotted and it is shown the non-slotted model is more optimistic than the slotted one contradictory to what is believed in the literature. By modifying the non-slotted model, the effect of the preamble on the channel capacity of IEEE 802.3 was studied under different regions of loads and different end-to-end propagation delays and its effect on performance degradation was analyzed.

Appendix

I- the original SMV model:

(1) Transition probabilities

$$p_{01} = 1 \quad \text{(A-1)}$$

$$p_{10} = e^{-G(1+a)} + \frac{1}{2} e^{-G(a+b)} (1 - e^{-2aG}) \quad \text{(A-2)}$$

$$p_{11} = Ge^{-G(1+a)} + \frac{1}{4} e^{-G(a+b)} .\{(1 - e^{-2aG})[1 + 2G(a+b)] - 2aGe^{-2aG}\} \quad \text{(A-3)}$$

$$p_{20} = e^{-G(a+b)} \quad \text{(A-4)}$$

$$p_{21} = G(a+b)e^{-G(a+b)} \quad \text{(A-5)}$$

(2) Stationary probabilities

$$\pi_1 = \frac{p_{20} + p_{21}}{k} \quad \text{(A-6)}$$

$$\pi_2 = \frac{1 - p_{10} - p_{11}}{k} \quad \text{(A-7)}$$

$$\pi_0 = 1 - \pi_1 - \pi_2 \quad \text{(A-8)}$$

$$k = (1 - p_{10} - p_{11})(1 + p_{20}) + (1 + p_{10})(p_{20} + p_{21}) \quad \text{(A-9)}$$

(3) States average times

$$E(T_0) = 1/G \quad \text{(A-10)}$$

$$E(T_1) = (1 - e^{-aG})(2a + b + 1/G) + e^{-aG} \quad \text{(A-11)}$$

$$E(T_2) = 2a + b \quad \text{(A-12)}$$

II-SMV with gap model

When we consider the effect of interfarme gap (g), the duration of state 1 will be larger by g than that of SMV without gap. The same is true for state 2. Thus:

$$p_{01} = 1 \quad \text{(A-13)}$$

$$p_{10} = e^{-G(1+a+g)} + \frac{1}{2} e^{-G(a+b+g)} (1 - e^{-2aG}) \quad \text{(A-14)}$$

$$p_{11} = G(1+g)e^{-G(1+a+g)} + \frac{1}{4} e^{-G(a+b+g)} .\{(1 - e^{-2aG})[1 + 2G(a+b+g)] - 2aGe^{-2aG}\} \quad \text{(A-15)}$$

$$p_{20} = e^{-G(a+b+g)} \quad \text{(A-16)}$$

$$p_{21} = G(a+b+g)e^{-G(a+b+g)} \quad \text{(A-17)}$$

The stationary probabilities can be calculated using the same relations between stationary and transition probabilities of the original SMV model. The states average times are given by:

$$E(T_0) = 1/G \quad \text{(A-18)}$$

$$E(T_1) = (1 - e^{-aG})(2a + b + 1/G) + e^{-aG} + g \quad \text{(A-19)}$$

$$E(T_2) = 2a + b + g \quad \text{(A-20)}$$

References

[1] F.A. Tobagi and V.B. Hunt, "Performance Analysis of Carrier Sense Multiple Access with Collision Detection," Computer Networks, Vol. 4, No. 5, pp. 245-259, Oct./Nov. 1980.

[2] K. Shoraby, M.L. Mole and A.N. Venetsanopoulos," Comments on Throughput Analysis for Persistent CSMA Systems," IEEE Trans. Commun. Vol. COM-35, pp. 240-243, Feb. 1987.

[3] T.A. Gonsalves, "Measured Performance of the Ethernet," in Advances in Local Area Networks, edited by K. kummerle, F.A. Tobagi and J.O. Limb, pp. 383-410, IEEE press, 1987.

[4] Gee Swee Poo and Tan Tiow Sena, "Performance Characteristics of a Broadband PC LAN: An Experimental Study,"computer communications, Vol. 10, No. 5, pp. 243-249, Oct. 1987.

[5] W. Bux, "Local-area subnetworks: A performance comparison," IEEE Trans. Cornmun., Vol. COM-29, pp. 1465-1473, Oct. 1981.

[6] Chikara Yasuda, Chikato Fujiwara and Takaaki Okumoto, "An Adaptive CSMA/CD System," Electronics and Communications in Japan, Part 1, Vol. 74, No. 7, 1991.

[7] Digital Equipment Corporation, Intel Corporation, Xerox Corporation, "The Ethernet: A Local Area Network: Data Link Layer and Physical Specifications," version 1, 1980.

[8] IEEE standard, "Part 3: Carrier sense multiple access with collision detection (CSMA/CD) access method and physical layer specifications" March 2002.

[9] 82586 Local Communications Controller, Intel Corporation 1982.

[10] Seo-Kyu Kweon and Kang G. shin, "Statistical Real-Time Communication over Ethernet," IEEE Trans. On Parallel and Distributed Systems, Vol. 14, No. 3, Mrach 2003.