A Comment on the Throughput of Non-persistent CSMA

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Abstract- A better estimate of the throughput of non-persistent CSMA without collision detection protocol is obtained by using a bus-star model. A bus when a station initiates transmission and a star when the transmission ends.

Keywords: non-persistent CSMA, bus model, throughput, non-homogenous Poisson process, infinite population model.

I. Introduction

The traditional models of the non-persistent CSMA without collision detection protocol assume that the network topology is star [1, 2, 3]. So when a station transmits a packet, it is possible for all the other stations to initiate transmissions within a vulnerable period equal to the end-to-end propagation delay. Also when the last transmitting station (assuming one or more stations are transmitting) ceases transmission, no station is allowed to transmit until the elapse of the vulnerable period (end-to-end propagation delay). This is unrealistic and gives a worst-case performance for the following two reasons:

1. In the bus model when a station initiates a transmission, then the number of stations that can send messages that could collide with that transmission decays with time. Hence, the probability of having a successful transmission is higher than in the worst-case star topology. The success probability is even higher when a station senses the channel idle and initiates a transmission while another transmission is in progress and still wiping the channel.
2. The idle time in the bus topology is less than the star one due to the possibility for a station to start transmission while another transmission is in progress and still wiping the channel without the need to wait end-to-end propagation delay.

In this paper, we use a bus-star model to capture one aspect of the bus-wise work of the non-persistent CSMA protocol. More advanced models were introduced in the literature using space time models [4, 5], but the approach here is new and the simplicity of the proof is worth to be considered.

II. Model

Consider an infinite population of stations, using non-persistent CSMA, is distributed uniformly on a bus of unit length. Each station generates traffic at an infinitesimally small rate, so that the total bus traffic sums to G packets per packet time. When a station initiates a transmission a bus topology is used so nearby stations will sense the channel busy before far ones (no need to wait the end-to-end propagation delay by all the stations), but when that station finishes its transmission a star topology is used so all stations need to wait end-to-end propagation delay before sensing the channel idle. It is clear the performance of this model is worse than the real bus model but better than the worst case star topology. Assume the packets are of constant duration, and the end-to-end propagation delay (normalized to the packet length) is a .

III. Non-persistent CSMA throughput

It is clear that our model differs in the probability of success from the star model in [1, 2, 3] and the busy period is smaller (see Appendix A) and the idle period is the same (using the same terminology introduced in [1, 2, 3]). The probability of success is given by the following theorem.

Theorem: Using the model in (II) the probability of a successful transmission is given by:

$$2\int_{0}^{1/2} e^{-Ga\frac{(1-2x^2)}{2}}dx$$

First Proof:

Divide the bus into n equal parts; hence each part will generate traffic with a rate of G/n (Fig. 1). Suppose now, the station at the most left hand side of the bus starts transmitting a packet. Imagine the packet to move in jumps from 0 to 1 to 2 and so on. This transmission will be successful if the packet while jumping from 0 to 1, no station right to 1 initiates a transmission; and while the packet jumps from 1 to 2, no station right to 2 initiates a transmission and so on until the packet reaches the most right hand side of the bus. The probability of the first event is $e^{-G(1-1/n)a/n}$, of the second event is

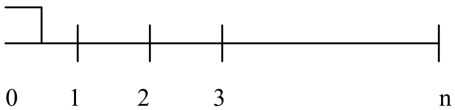

Fig. 1. Dividing the bus.

$e^{-G(1-2/n)a/n}$ and so on. Since all these events are independent, then we have:

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\begin{aligned}
P^{s0} &= \lim_{n\to\infty}\prod_{i=1}^{n} e^{-G(1-i/n)a/n} \\
&= \lim_{n\to\infty} e^{-\sum_{i=1}^{n} G(1-i/n)a/n} \\
&= e^{-Ga/2} \qquad (1)
\end{aligned}
$$

Where P^{sx} is the probability of successful transmission by a station at distance x from the left hand side of the bus. Now, assume a station at distance x from the left hand side starts transmission where $0 \le x \le ½$, it is clear the generation rate of the allowed stations to transmit will decay as twice as the above case as the signal propagates right and left from the station for a distance x and then the generation rate keeps decaying at the same rate as the above case (x=0) for a distance of 1-2x. Thus, by using the same approach above, we have:

$P^{sx} = \lim_{n\to\infty}\prod_{i=1}^{nx} e^{-G(1-2i/n)a/n}$ *. Pr. {no transmission as the packet propagates the right 1-2x of the bus}*[1] (2)

It is clear that the second term Pr.{…}= P^{s0} but with a propagation delay equals to (1-2x) a . So,

$$P^{sx} = e^{-Ga\frac{(1-2x^2)}{2}} \qquad (3)$$

It is clear that p^{sx} is symmetrical around x= ½ and with a maximum value at x= ½ and a minimum value when x=0= a . So,

Probability of successful transmission

$$= \int_0^1 p^{sx}dx = 2\int_0^{1/2} p^{sx}dx \qquad (4)$$

This is true because each station is equally likely to be the first transmitter.

Second proof:

Here, a simpler proof is given. Let a station at distance x, $0 \le x \le ½$, from the left hand side of the bus starts transmission. Then, as the signal propagates, the generation rate of the stations laying right and left of the packet edge is G(1-2t/ a) until the packet edge reaches the left hand side of the bus and G(1-t/ a) as the signal continues propagation until the right hand side of the bus; where t is the time, $0 \le t \le a$. So the packet generation process is a non-homogenous Poisson process [6] with a generation rate given by:

$$\lambda(t) = \begin{cases} G(1-2t/a) & 0 \le t \le ax \\ G(1-t/a) & ax < t \le (1-x)a \end{cases} \qquad (5)$$

The process mean is given by

$$
\begin{aligned}
m(t) &= \int_0^t \lambda(s)ds \\
&= \begin{cases} G(t-t^2/a) & 0 \le t \le ax \\ G(t-t^2/2a) - Ga\dfrac{x^2}{2} & ax < t \le (1-x)a \end{cases}
\end{aligned} \qquad (6)
$$

Success probability =Pr. {Zero arrivals in $[0,(1-x)a$]} $= e^{-[m((1-x)a)-m(0)]}$

The remainder of the proof is the same as the first one.

This completes the proof of the theorem. Now by using the same expressions for the busy[2] and ideal periods of the star model as derived in [1, 2, 3] we have the bus topology throughput is given by:

$$S_{bus} = \frac{G.2\int_0^{1/2} e^{-Ga(1-2x^2)/2}dx}{G(1+2a)+e^{-Ga}} \qquad (7)$$

The throughput equation derived in [1, 2, 3] is presented below for comparison:

$$S_{star} = \frac{Ge^{-Ga}}{G(1+2a)+e^{-Ga}} \qquad (8)$$

Both equations (7) and (8) were drawn in Fig. 2, where a significant increase in the channel capacity was obtained in the bus model.

Appendix A

In this Appendix we prove that the busy period of our model is smaller than the star model busy period derived in [1, 2, 3]. The busy period is given by 1+E(Y)+ a , where Y is the time between first transmission and the last transmission in the vulnerable period (see Fig. 3) and E(Y) is the expected value of Y. Now,

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[1] If nx is not an integer as when x is irrational, we can use [nx] without loss of generality as the effect diminishes when n goes to infinity.

[2] Since the busy period of the bus-star model is smaller than the star model busy period, then the throughput derived is a lower bound and hence the throughput of the bus-star model is higher.

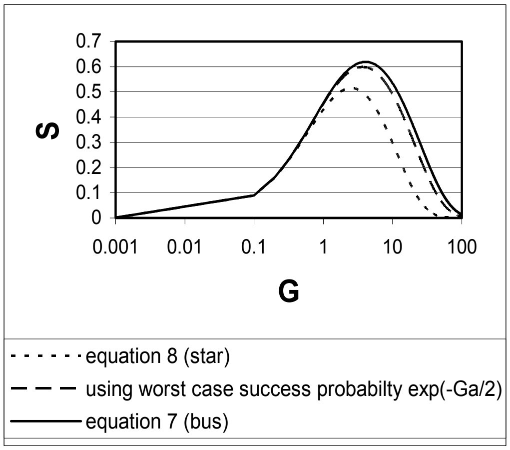

Fig. 2. Throughput as a function of the load, a =0.1.

$$E(Y)=\int_0^\infty \Pr.(Y>y)dy-\int_0^\infty \Pr.(Y<-y)dy$$
$$=\int_0^a (1-F_Y(y))dy$$

where $F_Y(y)$ =Pr. $\{Y\leq y\}$ is the distribution function. It is clear that $F_Y(y)$=Pr. {zero arrivals in a -y} and it is larger for our model than the star model because we have a smaller generation rate (due to the decay in the number of allowed stations to transmit as the signal propagates) in our model than the star model. Thus, E(Y) is smaller in our model than that of the star model and accordingly the busy period is smaller.

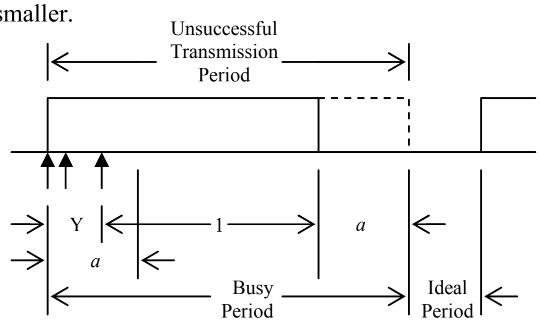

Fig. 3. Non persistent CSMA busy and ideal periods.

Appendix B

In proof one we implicitly used $p(\bigcap_{i=1}^{\infty} A_i)=\lim_{n\to\infty} p(\bigcap_{i=1}^{n} A_i)$ where A_i, i=1,2,3…etc are independent events. Indeed we have $p(\bigcap_{i=1}^{\infty} E_i)=\lim_{n\to\infty} p(E_n)$ for a decreasing sequence $E_1 \supset E_2 \supset E_3 ...etc.$ [7]. Now, Let E_1=A_1, E_2=A_1A_2, E_3=$A_1A_2A_3$ and so on. Then it is clear the sequence is decreasing and we have $p(\bigcap_{i=1}^{\infty} A_i)=p(\bigcap_{i=1}^{\infty} E_i)=\lim_{n\to\infty} p(E_n)=\lim_{n\to\infty} p(\bigcap_{i=1}^{n} A_i)$.

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