

Analytical solution of problems in heat transfer for pyramidal spines using Computer Algebra

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Abstract: The solution of the equation of heat transfer for pyramidal spines is obtained using computer algebra software. We consider pyramidal spines with three different sectional areas: triangle, square and pentagon. The solutions are given in terms of Bessel functions. From the temperature profiles that were obtained it is possible to deduce explicit formulas for the efficiency of the spine. The results that we derive here can be applied to the design of cooling systems of electronic circuits.

Keywords: conduction, convection, pyramidal spine, Bessel functions, CAS.

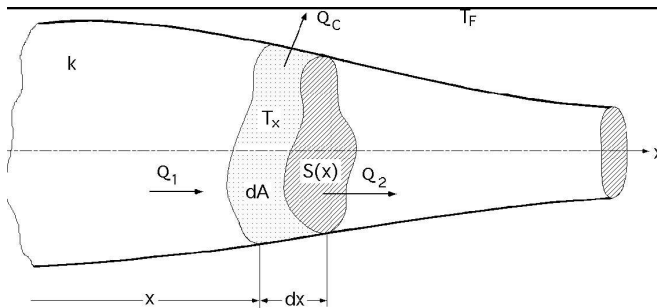
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1. Introduction

Due to the development and the importance of the usage of spines through heat transfer, different fields have been worked in such as Chemical Engineering and Mechanical engineering. Defining the spines (also known as Flippers), they can be defined as tubes in the form of flippers that allow to dissipate great amounts of heat that would be conformed by an additional surface to increase the heat transfer between a structure and a fluid. This is used in different technological applications such as refrigerators and cooling material.

It's important to mention, that the spines are divided into 3 groups. One of these groups will be our focus, which is the group of a variable section. The other 2 groups which are: constant section and other spines will be developed in another occasion.

In the variable section we will study and mention 3 kinds of geometrical figures: Square, Triangle and the Pentagon. A figure that will give us an idea of the equation model of the flipper that will be developed in the 3 cases of geometrical figures is shown below:



Q_1 is the heat entering in x , by conduction.

Q_2 is the outgoing heat in $(x + dx)$, by conduction

Q_C is the dissipated heat by conduction convection in the differential element dx

As we can see, in solids, the only method of heat transfer is conduction. If an end of a metallic spine is heated to gain temperature, the heat is transmitted to the colder end by conduction.

The proportionality factor is called thermal conductivity of the material. Materials such as gold, silver and copper have elevated thermal conductivities and conduct heat very well. On the other hand, materials

such as glass or leather have conductivities hundreds or a thousand times less, and have very bad heat conduction; these materials are also called insulating material.

In this study case, we are talking about a spine that has as a metal material that allows the movement of electrons in order to transfer heat in an adequate way. We can see in the figure above, that the spine is divided in segments of differential dx , that when are passing by Q2 produce $x+dx$ and is done by conduction (where heat flows from the coldest end to the hottest one) and finally QC is the dissipated heat by convection in the differential element dx , which means that if a difference in temperature exists in the interior of a liquid or a gas, its almost certain that a movement of the fluid will occur. This movement transfers heat from one part of the fluid to another by a process called convection. According to the above, we established enough conditions to model the spine equation, where it will only depend in space and not in time and we will relate to it simply with frontier conditions.

2. Model

According to this figure, let's proceed to the equation that will take us the development of the problem. The equation is [1], [2] and [5]:

$$\frac{d}{dx} \left[A_c(x) \frac{dT_{fin}(x)}{dx} \right] - \frac{h(x)}{k} \frac{dS(x)}{dx} [T_{fin}(x) - T_{\infty}] \equiv 0 \quad (1)$$

Where:

$A_c(x)$: is the area of the transversal section of the spine.

$\frac{dS(x)}{dx}$: Is the perimeter of transversal section.

dx : is the differential element.

The term T in function of x represents the temperature that enters and leaves the spine.

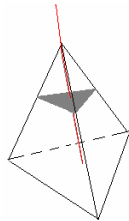
$h(x)$: is a constant

k : is a constant

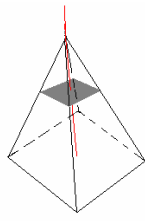
Like $A_c(x)$ represents the area of the cross section of the pyramid, we will show the expression that will result by representing the area by the triangle, the square and the pentagon and in that same way we will

show by the case of $\frac{dS(x)}{dx}$: which will be replaced in the original differential equation, but first we

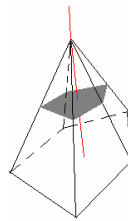
will show the 3 geometric figures that will be developed:



Triangle



Square



Pentagon

2.1 THE TRIANGLE: where

$$A_c(x) = \frac{\sqrt{3}}{4} (a^2 - 2a\beta x + \beta^2 x^2) \quad (2)$$

The expression β , will be defined by: $\beta = \frac{a}{b}$ a, b represent the sides of one portion of the triangle and

x is a variable in which the transversal section is involved towards the tip.

And the term:

$$\frac{dS(x)}{dx} = 3(a - \beta x) = 3L(x) = \textit{perimeter} \quad (3)$$

2.2 THE SQUARE: Where

$$Ac(x) = \left(\frac{a}{b}(b-x)\right)^2 \quad \text{or} \quad Ac(x) = a^2 - 2\frac{a^2x}{b} + \frac{a^2x^2}{b} \quad (4)$$

And the term:

$$\frac{dS(x)}{dx} = 4\left(\frac{a}{b}(b-x)\right) = 4l(x) = \text{perimeter} \quad (5)$$

2.3 THE PENTAGON

$$A = \frac{5a^2}{4} \cot \frac{\pi}{5} = \frac{a^2}{4} \sqrt{25 + 10\sqrt{5}} \cong 1.72048a^2 \quad (6)$$

$$\frac{dS(x)}{dx} = 5L(x) \quad (7)$$

3. Methods

For the method two development conditions can be established:

First, take the equation and replace it with the area of the desired geometrical figure, and after it's replaced, its factorized as much as possible. Then proceed to change the variable until a differential equation of second order is obtained and finally change another variable that will allow obtaining the general form of the Bessel function or simply taking the Maple [4] and putting the corresponding equation and having the program solve the problem until the Bessel functions [3] appear.

4. Results

The results that were obtained using certain algorithm for computer algebra are the following

4.1 The Square:

For the case of square pyramid the temperature profile is of the form

$$T(x) = - \frac{\text{BesselJ}\left(1, 4\sqrt{-h}\sqrt{\frac{a-\beta x}{\beta^2 k}}\right)\sqrt{a}(T_o - T_1)}{\sqrt{a-\beta x}\text{BesselJ}\left(1, 4\sqrt{-h}\sqrt{\frac{a}{\beta^2 k}}\right)} + T_o \quad (8)$$

4.2 The Triangle:

Now, for the case of triangular pyramid the temperature distribution is given by

$$T(x) = - \frac{\text{BesselJ}\left(1, 4\sqrt[3]{3}\sqrt{h}\sqrt{\frac{-a+\beta x}{\beta^2 k}}\right)\sqrt{-a}(T_o - T_1)}{\sqrt{-a+\beta x}\text{BesselJ}\left(1, 4\sqrt[3]{3}\sqrt{h}\sqrt{-\frac{a}{\beta^2 k}}\right)} + T_o \quad (9)$$

4.3 The pentagon:

Finally for the pentagonal pyramid the temperature profiles is

$$T(x) = - \frac{\text{BesselJ}\left[1, \frac{250 \sqrt{-2 h} \sqrt{10753} \sqrt{\frac{a - \beta x}{\beta^2 k}}}{10753}\right] \sqrt{-a} (T_o - T_1)}{\sqrt{\beta x - a} \text{BesselJ}\left[1, \frac{250 \sqrt{-2 h} \sqrt{10753} \sqrt{\frac{a}{\beta^2 k}}}{10753}\right]} + T_o \quad (10)$$

where $\text{BesselJ}(1,x) = J_1(x)$ and J_1 is the Bessel function of the first order.

From these results it is possible to compute the efficiency of the pyramidal spine according with the formulas given at [1], [2].

5. Discussion and conclusions (Technological and company applications)

According to what has been explained regarding the spines or flippers lets try comment which would be the technological applications:

One of the applications would be, in the household refrigerators. In the electrical ones, the heat produced in the condenser is dissipated by radiators that are placed in and outside the refrigerators frame. In the gas refrigerators the flippers are placed directly in the condenser.

Another application is in the portable refrigerators, where hot and cold foods can be stored. Since spines work by conduction and convection, its necessary to place 2 flippers that will allow each one to do the refrigeration process and therefore the refrigerated food is placed in the cold part and the same will apply for the other foods that don't need cold. Adding more to the problem, since the spines work with electricity, a kind of battery (light and portable) or lid should be used to indicate when the spines stop working. This of course, with the purpose of taking the foods out of the refrigerator when the spines stop working. (The flippers should always have a stable temperature, but not a constant one).

Another application is in the electric stoves; where we can clearly note a flipper working by convection and conduction. For example, by heating up a cup of water, the first element to work is the grill that transfers heat, from the coldest end to the hottest end through conduction and finally when the water is heated, it's done by convection

Another application of the spines is given in the refrigerator used with compression. The refrigerating element is evaporated taking its heat from the cold compartment and its heat is given to the exterior ambient when compressed to make it condense and lose heat in the condenser's tubes, which will turn cold and will transmit this to the food inside the refrigerator.

6. References

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