DESIGN OF A COMPUTER ALGEBRA ALGORITHM FOR ANALYTICAL SOLUTION AND CHARGE TRANSPORT INSIDE A CYLINDRICAL PLASMA

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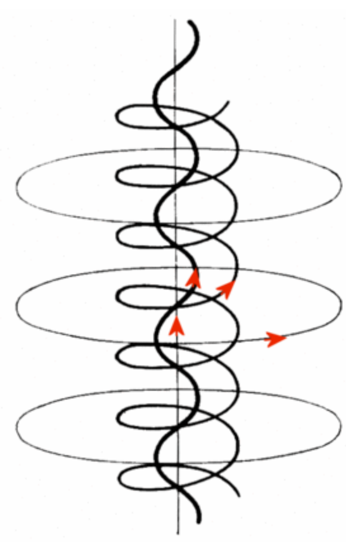
ABSTRACT
The symbolic computational analysis is an ideal tool for physical engineers, the power to intervene physical systems starting from the equations of the Mathematical Physics, like in this case in which is determined analytically the density of the ions or electrons N(r,t) in a confined plasma in a cylindrical cavity and the critical radius for the existence it gives. All of this was implemented by an algebra computational algorithm which mathematical structure is the technical Laplace transform.

Keywords: Plasma, Fick' law, Laplace transform, Bromwich Integration, CAS.

INTRODUCTION

At the present time the Plasm is used in multiple applications; standing out the coatings of materials, the optic systems and the investigation in Plasm reactors (nuclear fusion). There are different forms for generating plasma. In general, it consists on a reactor by means of which a gas is put under a high potential difference (of the order of 400 Watt). The plasma is defined as an ionized gas, with a net load which is almost neutral and with a collective behaviour; the collective behaviour means that the electromagnetic interactions (from long reach) determine the statistical properties of the system **[1]**. Quantum mechanical laws govern the Plasma, but this implies a high complexity. Even more, we intend that transport study of the Plasma load can be studied from Fick's law; considering the case of the non homogeneous equation, that is to say the existence of an electromagnetic field **[2]**.

The plasma was generated by a reactor of microwaves and then redirected to a cylindrical cavity in which it was confined. The obtained equation for the study is



$$\left(\frac{\partial}{\partial t}\mathrm{N}(r,t)\right)-\frac{H\left(\left(\frac{\partial}{\partial r}\mathrm{N}(r,t)\right)+r\left(\frac{\partial^2}{\partial r^2}\mathrm{N}(r,t)\right)\right)}{r}=K\,\mathrm{N}(r,t) \qquad (1)$$

With the initial condition:

$$N(r,0) = 0 \tag{2}$$

And with the boundary conditions

$$N(a, t) = N_o \mathbf{e}^{(-\beta t)} \tag{3}$$

Where **N(r,t)** represents the density of ions or electrons at a distance **r** from the axis of the cylinder at time **t**; **H** is the diffusion constant of the ions or electrons; **K** is the effective constant of chain ionization or autocatalytic; **β** is the decline constant from the ionization at frontier **r=a** and $\mathbf{N_o}$ is the initial density of ions in the frontier. The equation (1) was constructed by considering Fick's law, the geometry of the cavity and the Debye length (L>>a), which refers to the minimal distance at which the group of particles continues interacting as a plasma **[1]**. Here we assume that the process of ionization in the initially non ionized gas is generated by diffusion of ions from the frontier in r=a. The frontier ions are considered to be produced by some ionization mechanism.

METHOD

With the aim to solve the equation (1) and with the conditions (2) and (3), a certain algorithm of computer algebra was constructed. In such an algorithm, the Laplace transform technique was applied, and the Inverse Laplace transform was realized using the Bromwich's integral and residue calculus **[3]**. The algorithm of computer algebra that was used can be presented as:

1. Apply the Laplace Transform to system (1) with (2) in order to turn the partial equations into ordinary differential equations.
2. Solve such system of ordinary equations using condition (3).
3. Realize the Inverse Laplace Transform using the Bromwich's integral and residue theorem.
4. Extract from the solution the form of the transport charge in plasma with boundary condition.

Full details of the step by step method of this algorithm are presented in the Appendix. For the development of the problem Maple 9 **[4]** was used.

RESULTS

The analytical solution (4) allows us to analyze the most decisive transport variables of the load in plasma, and also enables us to determine the critical radius for the existence of the plasma (5).

$$N(r, t) = \frac{N_1 J_0\left(\sqrt{\frac{\beta + K}{H}}\, r\right)}{\mathbf{e}^{(\beta t)} J_0\left(\sqrt{\frac{\beta + K}{H}}\, a\right)} + \left(\sum_{n=1}^{\infty}\left(\frac{2 J_0\left(\frac{\alpha_n r}{a}\right) \alpha_n N_1 H \mathbf{e}^{\left(\frac{\left(K a^2 - \alpha_n^2 H\right) t}{a^2}\right)}}{(K a^2 - \alpha_n^2 H + \beta a^2)\, J_1(\alpha_n)}\right)\right) \tag{4}$$

Where J_o and J_1 are the Bessel functions of the first kind, the zero and first order, respectively. They satisfy Bessel's equation.

α_n represents the roots of the Bessel function J_o it is to say are the solutions of the equations $J_0(x) = 0$ **[5] .**

Based on (4) the critical radius is given by:

$$a_c = \alpha_n \sqrt{\frac{K}{H}} \tag{5}$$

CONCLUSIONS

With the vital tool of the symbolic computational analysis, it was possible to determine the critical radius for the formation and also the permanency in the plasma time, as well as it allowed to differ and to vary the dimensions of the more significant contributions for the conservation of the plasma.

In the same way, the determination of the critical radius is a clear example of how the used technique of symbolical computation allows to intervene directly in physical systems and to make an exact and effective use of the Mathematical Physics equations, in order to analyze, describe and propose solutions.

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APPENDIX

The initial condition indicates that in time zero the density of load is zero; taking the Laplace transform, solving and replacing gives us

$$\Phi(r) = \_C1 \text{ BesselJ}\left(0, \sqrt{\frac{-s+K}{H}}\, r\right) + \_C2 \text{ BesselY}\left(0, \sqrt{\frac{-s+K}{H}}\, r\right) \tag{6}$$

Because the BesselY function diverges for r =0, and as the solution should be finite in zero, C2 is chosen = 0; therefore (6) reduces to

$$\Phi(r) = \_C1 \text{ BesselJ}\left(0, \sqrt{\frac{-s+K}{H}}\, r\right) \tag{7}$$

Solving (7) for the boundary condition (3) we get

$$\Phi(r) = \frac{N_1 \text{ BesselJ}\left(0, \sqrt{\frac{-s+K}{H}}\, r\right)}{\text{BesselJ}\left(0, \sqrt{\frac{-s+K}{H}}\, a\right)(s+\beta)} \tag{8}$$

In order to return to the domain of interest (real) we use the Bromwich's integrate (8), to calculate Laplace's inverse transform

$$F(s) := \int_0^\infty e^{(-s\,t)} f(t)\, dt \tag{9}$$

Where F, from the complex variable is analytical in the whole plane, except in a finite number of isolated singularity points. To calculate this integral we use the theorem of the residuals. For that, we use the poles of the function (8), which possesses one pole of order 1 in ($s = -\beta$) and infinite poles where the Bessel function goes to zero.

For $s = -\beta$ we obtained the following residual

$$Residuo1 := \frac{N_1 J_0\left(\sqrt{\frac{\beta+K}{H}}\, r\right)}{e^{(\beta\, t)} J_0\left(\sqrt{\frac{\beta+K}{H}}\, a\right)} \tag{10}$$

Then we obtain the poles for the Bessel function for which we chose by means of the systemic determination

$$Polos1 := \sqrt{\frac{-s+K}{H}}\, a = \alpha_n \qquad \text{Therefore,} \qquad s = -\frac{-K a^2 + \alpha_n^{\,2} H}{a^2} \tag{11}$$

And the following residual was obtained

$$\sum_{n=1}^{\infty} \left(\frac{2 J_0\left(\frac{\alpha_n r}{a}\right) \alpha_n N_1 H e^{\left(\frac{(K a^2 - \alpha_n^{\,2} H)\, t}{a^2}\right)}}{(K a^2 - \alpha_n^{\,2} H + \beta a^2)\, J_1(\alpha_n)} \right) \tag{12}$$