Evolutionary Optimization of Interval Type-2 Membership Functions

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Abstract—Uncertainty is an inherent part in controllers used for real-world applications. The use of new methods for handling incomplete information is of fundamental importance in engineering applications. We simulated the effects of uncertainty produced by the instrumentation elements in type-1 and type-2 fuzzy logic controllers to perform a comparative analysis of the systems' response, in the presence of uncertainty.
We are presenting an innovative idea to optimize interval type-2 membership functions, we are showing comparative results of the optimized proposed method. We found that the optimized membership functions for the inputs of a type-2 system increases the performance of the system for high noise levels.

Keywords. Interval type-2 fuzzy logic, evolutionary algorithm, fuzzy control.

1. Introduction

Uncertainty affects decision-making and appears in a number of different forms. The concept of information is fully connected with the concept of uncertainty. The most fundamental aspect of this connection is that the uncertainty involved in any problem-solving situation is a result of some information deficiency, which may be incomplete, imprecise, fragmentary, not fully reliable, vague, contradictory, or deficient in some other way [1]. The general framework of fuzzy reasoning allows handling much of this uncertainty, fuzzy systems employ type-1 fuzzy sets, which represents uncertainty by numbers in the range [0, 1]. When something is uncertain, like a measurement, it is difficult to determine its exact value, and of course, type-1 fuzzy sets makes more sense than using crisp sets [2]. However, it is not reasonable to use an accurate membership function for something uncertain, so in this case what we need is another type of fuzzy sets, those which are able to handle these uncertainties, the so called type-2 fuzzy sets [3]. So, the amount of uncertainty in a system can be reduced by using type-2 fuzzy logic because it offers better capabilities to handle linguistic uncertainties by modeling vagueness and unreliability of information [4].

Recently, we have seen the use of type-2 fuzzy sets in fuzzy logic systems to deal with uncertain information [5,6]. So we can find some papers emphasizing on the implementation of a type-2 Fuzzy Logic System (FLS); in others, it is explained how type-2 fuzzy sets let us model and minimize the effects of uncertainties in rule-base FLSs [7]. Some research works are devoted to solve real world applications in different areas, for example, in signal processing type-2 fuzzy logic is applied in prediction in Mackey-Glass chaotic time-series with uniform noise presence [8]. In medicine, an expert system was developed for solving the problem of Umbilical Acid-Base (UAB) assessment [9]. In industry, type-2 fuzzy logic and neural networks was used in the control of non-linear dynamic plants [10, 11].

This work deals with the optimization of interval type-2 membership functions in a fuzzy logic controller (FLC). It is a fact, that in the control of real systems, the instrumentation elements (instrumentation amplifier, sensors, digital to analog, analog to digital converters, etc.) introduce some sort of unpredictable values in the information that has been collected. So, controllers designed under idealized conditions tend to behave in an inappropriate manner. Since, uncertainty is inherent in controllers for real world applications, as a first step, we are presenting how to deal with it using type-2 FLC to diminish the effects of imprecise information. We are supporting this statement with experimental results, qualitative observations, and quantitative measures of errors. For quantifying the errors, we utilized three widely used performance criteria, these are: Integral of Square Error (ISE), Integral of the Absolute value of the Error (IAE), and Integral of the Time multiplied by the Absolute value of the Error (ITAE) [12].

Then as a second step, we optimized the parameters of the Gaussian membership functions (MFs) using an evolutionay method and ISE as the fitness function. In this case, we used as an output, the average of two type-1 system, and we made again the experiment.

This paper is organized as follows: Section 2 presents an introductory explanation of type-1 and type-2 FLCs and the performance criteria for evaluating the transient and steady state closed-loop response in a computer control system. Section 3, we describe the Human Evolutionary Method (HEM), to optimize the interval type-2 MFs

Section 4, is devoted to experimental results. In this section we are showing details of the implementation of the feedback control system used. We are presenting results from several experiments, the plant was tested using several signal to noise ratio, we are including a performance comparison between normal type-1 and type-2 fuzzy logic controllers versus optimized type-2 fuzzy logic controllers.

Finally, in section 5, we have the conclusions.

2. Fuzzy Controllers

A. Type-1 Fuzzy controller

Soft computing techniques have become a research topic, which is applied in the design of controllers [13]. These techniques have tried to avoid the above mentioned drawbacks, and they allow us to obtain efficient controllers, which utilize the human experience in a more related form than the conventional mathematical approach. In the cases in which a mathematical representation of the controlled systems cannot be obtained, the process operator should be able to express the relationships existing in them, that is, the process behavior.

A FLS, described completely in terms of type-1 fuzzy sets is called a type-1 fuzzy logic system (type-1 FLS). It is composed by a knowledge base that comprises the information given by the process operator in form of linguistic control rules; a fuzzification interface, who has the effect of transforming crisp data into fuzzy sets; an inference system, that uses them in conjunction with the knowledge base to make inference by means of a reasoning method; and a defuzzification interface, which translates the fuzzy control action so obtained to a real control action using a defuzzification method.

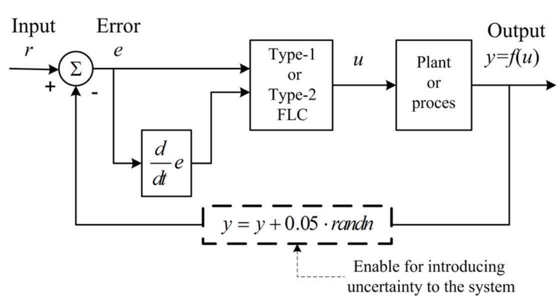

Fig.1. System used for obtaining the experimental results.

In our paper, the implementation of the fuzzy controller in terms of type-1 fuzzy sets, has two input variables such as the error $e(t)$, the difference between the reference signal and the output of the process, as well as the error variation $\Delta e(t)$,

$$e(t) = r(t) - y(t) \quad (1)$$

$$\Delta e(t) = e(t) - e(t-1) \quad (2)$$

so the control system can be represented as in Fig. 1.

B. Type-2 Fuzzy controller

If we have a type-1 membership function as in Fig. 2, and we blurring it to the left and to the right, then, at a specific value x', the membership function (u'), takes on different values which not all be weighted the same, so we can assign an amplitude distribution to all of those points.

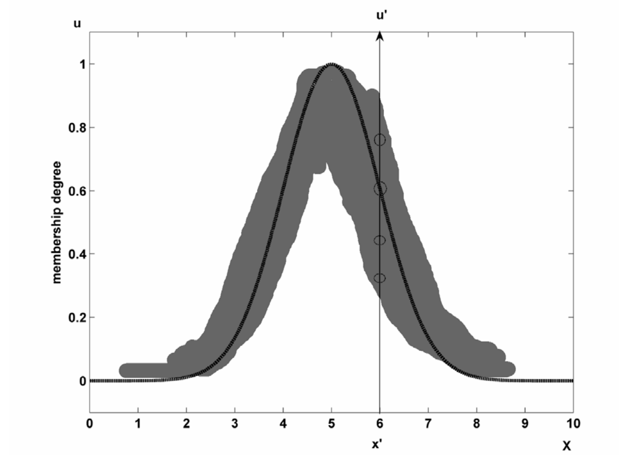

Fig. 2. Blurred type-1 membership function.

Doing this for all $x \in X$, we create a three-dimensional membership function –a type-2 membership function– that characterizes a type-2 fuzzy set [3,7]. A type-2 fuzzy set $\widetilde{A}$, is characterized by the membership function:

$$\widetilde{A} = \left\{ \left((x,u), \mu_{\widetilde{A}}(x,u) \right) \mid \forall x \in X, \forall u \in J_x \subseteq [0,1] \right\} \quad (3)$$

in which $0 \leq \mu_{\widetilde{A}}(x,u) \leq 1$. Another expression for $\widetilde{A}$ is,

$$\widetilde{A} = \int_{x \in X} \int_{u \in J_x} \mu_{\widetilde{A}}(x,u) / (x,u) \qquad J_x \subseteq [0,1] \quad (4)$$

Where $\int\int$ denote union over all admissible input variable x and u. For discrete universes of discourse $\int$ is replaced by $\sum$ [7]. In fact $J_x \subseteq [0,1]$ represents the primary membership of x and $\mu_{\widetilde{A}}(x,u)$ is a type-1 fuzzy set known as the secondary set. Hence, a type-2 membership grade can be any subset in [0,1], the primary membership, and corresponding to each primary membership, there is a secondary membership (which can also be in [0,1]) that defines the uncertainty for the primary membership.

This uncertainty is represented by a region called footprint of uncertainty (FOU). When $\mu_{\tilde{A}}(x,u)=1, \forall u \in J_x \subseteq [0,1]$ we have an interval type-2 membership function, as shown in Fig. 3. The uniform shading for the FOU represents the entire interval type-2 fuzzy set and it can be described in terms of an upper membership function $\overline{\mu}_{\tilde{A}}(x)$ and a lower membership function $\underline{\mu}_{\tilde{A}}(x)$.

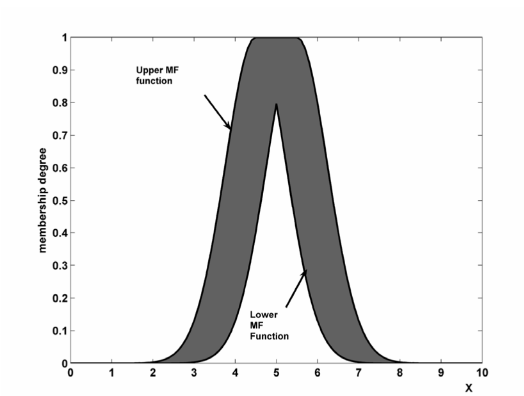

Fig. 3. Interval type-2 membership function

A FLS described using at least one type-2 fuzzy set is called a type-2 FLS. Type-1 FLSs are unable to directly handle rule uncertainties, because they use type-1 fuzzy sets that are certain [14]. On the other hand, type-2 FLSs, are very useful in circumstances where it is difficult to determine an exact certainty, and measurement uncertainties [3].

It is known that type-2 fuzzy sets let us to model and to minimize the effects of uncertainties in rule-based FLS. Unfortunately, type-2 fuzzy sets are more difficult to use and understand than type-1 fuzzy sets; hence, their use is not widespread yet. In [7] were mentioned at least four sources of uncertainties in type-1 FLSs:

1. The meanings of the words that are used in the antecedents and consequents of rules can be uncertain (words mean different things to different people).
2. Consequents may have histogram of values associated with them, especially when knowledge is extracted from a group of experts who do not all agree.
3. Measurements that activate a type-1 FLS may be noisy and therefore uncertain.
4. The data used to tune the parameters of a type-1 FLS may also be noisy.

All of these uncertainties translate into uncertainties about fuzzy set membership functions. Type-1 fuzzy sets are not able to directly model such uncertainties because their membership functions are totally crisp. On the other hand, type-2 fuzzy sets are able to model such uncertainties because their membership functions are themselves fuzzy. A type-1 fuzzy set is a special case of a type-2 fuzzy set; its secondary membership function is a subset with only one element, unity.

Type-2 FLSs, can be used when the circumstances are too uncertain to determine exact membership grades such as when training data is corrupted by noise. In our case, we are simulating that the instrumentation elements (instrumentation amplifier, sensors, digital to analog, analog to digital converters, etc.) are introducing some sort of unpredictable values in the collected information.

In the case of the implementation of the type-2 FLC, we have the same characteristics as in type-1 FLC, but we used type-2 fuzzy sets as membership functions for the inputs and for the output.

To optimized the interval type-2 membership functions, which we formed with two type-1 Gaussian MF with constant centers and different standard deviations, we established two type-1 FLC, each one with its inputs, output and membership functions. In this work, we used an innovative idea to optimize these interval type-2 membership functions, the Human Evolutionary Method (HEM) which we will explain in the next section. As a fitness function we took ISE, we maintained constant the centers of the Gaussian MFs and looked for the best values of the standard deviations. In experiment 6, we presented the results of this optimization.

Performance Criteria

For evaluating the transient closed-loop response of a computer control system we can use the same criteria that normally are used for adjusting constants in PID (Proportional Integral Derivative) controllers. These are [12]:

1. Integral of Square Error (ISE).

$$\mathrm{ISE} = \int_0^\infty [e(t)]^2\,dt \qquad (5)$$

2. Integral of the Absolute value of the Error (IAE).

$$\mathrm{IAE} = \int_0^\infty |\,e(t)\,|\,dt \qquad (6)$$

3. Integral of the Time multiplied by the Absolute value of the Error (ITAE).

$$\text{ITAE} = \int_0^\infty t\,|\,e(t)\,|\,dt \qquad (7)$$

The selection of the criteria depends on the type of response desired, the errors will contribute different for each criterion, so we have that large errors will increase the value of ISE more heavily than to IAE. ISE will favor responses with smaller overshoot for load changes, but ISE will give longer settling time. In ITAE, time appears as a factor, and therefore, ITAE will penalize heavily errors that occurs late in time, but virtually ignores errors that occurs early in time.

3. The Human Evolutionary Model

In this section, we are going to explain HEM [15], because it is the EA that we used to make the experiments. HEM is a "*Self-adaptive algorithm that evaluates its own behavior and changes its behavior when the evaluation indicates that it is not accomplishing what the algorithm is intended to do, or when better functionality or performance is possible*". Because solving single objective optimization (SOO) and multiple objective optimization (MOO) problems is different, HEM has two operational forms. In this work we are only dealing with SOO problems, hence the next explanation will be focus on solving SOO problems.

HEM has the skill of avoiding being trapped in the same region of the landscape, as well to promote the evolution towards optimal solution. HEM is an EA that has nine principal components, in the next expression we are showing them,

$$HEM = \left(H, AIIS, P, O, S, E, L, TL/PS, VRL\right)$$

where

H	Human
AIIS	Adaptive Intelligent Intuitive System
P	Population of size *N* human like individuals
O	Single or a multiple objective optimization goals
S	Evolutionary strategy used for reaching the objectives expressed in *O*
E	Environmental here we can have predators, etc.
L	Landscape, i.e., the scenario where the evolution must be performed
TL/PS	Tabu List formed by the best solution founded/Pareto Set for Multiobjective optimization
VRL	Visited Regions List

HEM is an intelligent EA that is learning from the experts their rational and intuitive procedures that they use to solve optimization problems. In this model, we are considering that we have two kinds of humans, one class is form by human beings, and the other class is an artificial human that has been implemented in AIIS of HEM. Humans as part of the system, they are in charge of teaching the artificial human, all the knowledge needed for realizing the searching task. The AIIS should learn the rational and intuitive knowledge from the experts; the final purpose is that the artificial human eventually can substitute the human beings in most of the times.

4. Experimental Results.

Figure 1 shows, the feedback control system that was used for achieving the results of this paper. It was implemented in Matlab where the controller was designed to follow the input as closely as possible.

The plant was modeled using equation (8)

$$y(i) = 0.2 \cdot y(i-3) \cdot 0.07 y(i-2) + 0.9 \cdot y(i-1) + 0.05 \cdot u(i-1) + 0.5 \cdot u(i-2) \qquad (8)$$

The controller's output was applied directly to the plant's input. Since we are interested in comparing the performance between normal type-1 and type-2 FLC system versus optimized type-2 FLC system, we tested the controller in three ways:

1. Considering the system as ideal, that is, we did not introduce in the modules of the control system any source of uncertainty. See experiments 1, and 2.
2. Simulating the effects of uncertain modules (subsystems) response introducing some uncertainty, and diverse noise levels. See experiments 3, 4 and 5.
3. After optimization of the interval type-2 MFs, we repeated case two above. See experiment 6.

For case one, as is shown in Fig. 1, the system's output is directly connected to the summing junction, but in the second case, the uncertainty was simulated introducing random noise with normal distribution (the dashed square in Fig. 1). We added noise to the system's output $y(i)$ using the Matlab's function "randn" which generates random numbers with Gaussian distribution. The signal and the added noise in turn, were obtained with the programmer's expression (9), the result $y(i)$ was introduced to the summing junction of the controller system. Note that in (9) we are using the value 0.05 for experiments 3 and 4, but in the set of tests for experiment 5 we varied this value to obtain different SNR values.

$$y(i) = y(i) + 0.05 \cdot randn \qquad (9)$$

We tested the system using as input, a unit step sequence free of noise, $r(i)$. For evaluating the system's response and compare between type 1 and type 2 fuzzy controllers, we used the performance criteria ISE, IAE, and ITAE. In Table II, we summarized the values obtained for each criterion considering 200 units of time. For calculating ITAE we considered a sampling time $T_s = 0.1$ sec.

For experiments 1, 2, 3, and 4 the reference input r is stable and noisy free. In experiments 3 and 4, although the reference appears clean, the feedback at the summing junction is noisy since we introduced deliberately noise for simulating the overall existing uncertainty in the system, in consequence, the controller's inputs $e(t)$ (error), and $\Delta e(t)$ contains uncertainty data.

In experiment 5, we tested the systems, type-1 and type-2 FLCs, introducing diverse values of noise η, this is modifying the signal to noise ratio SNR [16],

$$SNR = \frac{\sum |s|^2}{\sum |\eta|^2} = \frac{P_{signal}}{P_{noise}} \qquad (10)$$

Because many signals have a very wide dynamic range [17], SNRs are usually expressed in terms of the logarithmic decibel scale, SNR(db),

$$SNR(db) = 10\log_{10}\left(\frac{P_{signal}}{P_{noise}}\right) \qquad (11)$$

In Table IV, we show, for different values of SNR(db), the behavior of ISE, IAE, ITAE for type-1 and type-2 FLCs. In almost all the cases the results for type-2 FLC are better than type-1 FLC.

In type-1 FLC, we selected Gaussian MFs for the inputs and for the output. A Gaussian MF is specified by two parameters {c,σ}:

$$\mu_A(x) = e^{-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2} \qquad (12)$$

c represents the MFs center and σ determines the MFs standard deviation.

For each input of the type-1 FLC, $e(t)$ and $\Delta e(t)$, we defined three type-1 fuzzy Gaussian MFs: negative, zero, positive. The universe of discourse for these membership functions is in the range [-10 10]; their centers are -10, 0 and 10 respectively, and their standard deviations are 9, 2 and 9 respectively .

For the output of the type-1 FLC, we have five type-1 fuzzy Gaussian MFs: NG, N, Z, P and PG. They are in the interval [-10 10], their centers are -10, -4.5, 0, 4.5, and 10 respectively; and their standard deviations are 4.5, 4, 4.5, 4 and 4.5.

For the type-2 FLC, as in type-1 FLC we also selected Gaussian MFs for the inputs and for the output, but in this case we have an interval type-2 Gaussian MFs with a fixed standard deviation, σ, and an uncertain center, ie.,

$$\mu_A(x) = e^{-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2} \qquad c \in [c_1, c_2] \qquad (13)$$

In terms of the upper and lower membership functions, we have for $\bar{\mu}_{\tilde{A}}(x)$,

$$\bar{\mu}_{\tilde{A}}(x) = \begin{cases} N(c_1, \sigma; x) & x < c_1 \\ 1 & c_1 \le x \le c_2 \\ N(c_2, \sigma; x) & x > c_2 \end{cases} \qquad (14)$$

TABLE I

CHARACTERISTICS OF THE MFS OF THE INPUTS AND OUTPUT OF THE TYPE-2 FLC.

Variable	Term	Center c_1	Center c_2	Standard Deviation σ
Input e	negative	-10.25	-9.75	9.2
	zero	-0.25	0.25	2.2
	positive	9.75	10.25	9.2
Input Δe	negative	-10.25	-9.75	9.2
	zero	-0.25	0.25	2.2
	positive	9.75	10.25	9.2
Output cde	NG	-10.25	-9.75	4.5
	N	-4.75	-4.5	4
	Z	-0.25	0.25	4.5
	P	3.75	4.25	4
	PG	9.75	10.25	4.5

and for the lower membership function $\underline{\mu}_{\tilde{A}}(x)$,

$$\underline{\mu}_{\tilde{A}}(x) = \begin{cases} N(c_2, \sigma; x) & x \le \frac{c_1 + c_2}{2} \\ N(c_1, \sigma; x) & x > \frac{c_1 + c_2}{2} \end{cases} \qquad (15)$$

where $N(c_1, \sigma, x) \equiv e^{-\frac{1}{2}\left(\frac{x-c_1}{\sigma}\right)^2}$,

and $N(c_2, \sigma, x) \equiv e^{-\frac{1}{2}\left(\frac{x-c_2}{\sigma}\right)^2}$ [3].

Hence, in type-2 FLC, for each input we defined three interval type-2 fuzzy GaussianMFs: negative, zero, positive in the interval [-10 10]. For computing the output we have five interval type-2 fuzzy Gaussian MFs NG, N, Z, P and PG, with uncertain center and fixed standard deviations in the interval [-10 10]. Table I shows the characteristics of the MFs of the inputs and output of the type-2 FLC.

In experiment 6, to simulate the interval type-2 MFs of the FLC, we used two type-1 Gaussian MF with constant centers and different standard deviations. Using HEM as an optimized method, ISE as a fitness function, we found the best values of the parameters of the type-1 Gaussian MFs of the inputs of these controllers, see table IV. Through an average of the two type-1 optimized FLCs, we repeated experiment 5, and calculated again the values of ISE, IAE and ITAE, as can be seen in table V.

For the experiments with interval type-2 MFs not optimized, we used, basically, the free software available online [18].

Experiment 1. Ideal system using a type-1 FLC.
In this experiment, we did not add uncertainty data to the system. The system trends to stabilize with time and the output will follow accurately the input. In

Table II, we listed the obtained values of ISE, IAE, and ITAE for this experiment.

Experiment 2. Ideal system using a type-2 FLC.
Here, we used the same test conditions of Experiment 1, but in this case, we implemented the controller's algorithm with type-2 fuzzy logic. The corresponding performance criteria are listed in Table II

Experiment 3. System with uncertainty using a type-1 FLC.
In this case, we simulated using equation (9), the effects of uncertainty introduced to the system by transducers, amplifiers, and any other element that in real world applications affects expected values. In Table II, we can see the obtained values of ISE, IAE, and ITAE for a simulated 10 db signal noise ratio.

Experiment 4. System with uncertainty using a type-2 FLC. In this experiment, we introduced uncertainty in the system, in the same way as in Experiment 3. In this case, we used a type-2 FLC and we improved those results obtained with a type-1 FLC in Experiment 3, see table II.

TABLE II
COMPARISON OF PERFORMANCE CRITERIA FOR TYPE-1 AND TYPE-2 FUZZY LOGIC CONTROLLERS FOR 10 DB SIGNAL NOISE RATIO. VALUES OBTAINED AFTER 200 SAMPLES.

Performance Criteria	Type-1 FLC		Type-2 FLC	
	Ideal System	Syst. with uncertainty	Ideal System	Syst. with uncertainty
ISE	5.2569	205.0191	5.2572	149.3097
IAE	13.8055	155.9412	13.7959	131.77
ITAE	46.0651	1583.4	45.8123	1262.2

Experiment 5. Varying the signal to noise ratio in type-1 and type-2 FLCs.
To test the robustness of the type-1 and type-2 FLCs, we repeated experiments 3 and 4 giving different noise levels, going from 30 db to 6 db of SNR ratio in each experiment. In Table III, we summarized the values for ISE, IAE, ITAE considering 200 units of time with a P_{signal} of 22.98 db in all cases. As it can be seen in Table III, in presence of diverse noise levels, the behavior of type-2 FLC is better than type-1 FLC above 10 db.

From Table III, taking two examples, the extreme cases; we have for an SNR ratio of 8 db, in type-1 FLC the next performance values ISE=1004, IAE=352.45, ITAE=4526; for the same case, in type-2 FLC, we have ISE=903, IAE=330.38, ITAE=4104.

For 10 db of SNR ratio, we have for type-1 FLC, ISE=205.01, IAE=155.94, ITAE=1583.4, and for type-2 FLC, ISE=149.3, IAE=131.77, ITAE=1262.2.

TABLE III
BEHAVIOR OF TYPE -1 AND TYPE-2 FUZZY LOGIC CONTROLLERS AFTER VARIATION OF SIGNAL NOISE RATIO. VALUES OBTAINED FOR 200 SAMPLES.

SNR db	Type-1 FLC ISE	Type-1 FLC IAE	Type-1 FLC ITAE	Type-2 FLC ISE	Type-2 FLC IAE	Type-2 FLC ITAE
6	1208	392.3	4903	1113	368.7	4388
8	1004	352.4	4526	903	330.3	4104
10	205.0	155.9	1583.4	149.3	131.7	1262.2
12	89.77	102.1	974.97	89.8	102.2	974.93
14	56.47	80.88	769.51	56.78	80.85	770.25
16	36.28	64.36	610.86	36.39	64.21	610.65
18	23.76	51.54	485.19	23.81	51.32	485.16
20	16.14	41.75	386.45	16.04	41.5	386.6
22	11.36	34.65	310.04	11.25	34.28	308.87
24	8.54	29.25	249.67	8.39	28.78	247.89
26	6.87	25.15	202.46	6.72	24.78	201.07
28	5.9	22.2	166.38	5.78	21.92	165.21
30	5.38	20.12	139.27	5.27	19.77	137.47

These values indicate a better performance of type-2 FLC than type-1 FLC above certain noise values, because they are a representation of the errors and as bigger they are the performance of the system is worst

Experiment 6. Optimizing the interval type-2 MFs of the FLC.

To optimized the interval type-2 MFs of the FLC, we simulated that system using two type-1 FLCs . We maintain constant the centers of the Gaussian MFs of the inputs and varied its standard deviations. After using the HEM as the optimized method, and taking ISE as the fitness function, we found the best values of the MFs, as can be seen in table IV.

With the new values of the MFs of both type-1 FLCs, we repeated experiment 5, but in this case, we used the average of the two type-1 FLCs as the output of the type-2 system.

TABLE IV
CHARACTERISTICS OF THE OPTIMIZED MFS OF THE INPUTS AND OUTPUT OF THE TYPE-2 FLC.

Variable	Term	Center c_1	Standard Deviation σ_1	Center c_2	Standard Deviation σ_2
Input e	negative	-10	9	-10	8.0298
	zero	0	2	0	1.0987
	positive	10	9	10	8.1167
Input Δe	negative	-10	9.2	-10	8.7767
	zero	0	2.2	0	1.0987
	positive	10	9.2	10	8.5129
Output cde	NG	-10	4.5	-10	4.5
	N	-4.5	4	-4.5	4
	Z	0	4.5	0	4.5
	P	4	4	4	4
	PG	10	4.5	10	4.5

Table V, shows the results for this experiment, as can be seen, all the values of ISE were improved, and in general we can see that the performance of the system is better.

TABLE V
COMPARISON OF PERFORMANCE CRITERIA FOR TYPE-1 FLC, AND TYPE-2 FUZZY LOGIC CONTROLLER WITH OPTIMIZED MFS, FOR 10 DB SIGNAL NOISE RATIO. VALUES OBTAINED AFTER 200 SAMPLES.

SNR	**Type-1 FLC**			**Type-2 FLC**		
db	ISE	IAE	ITAE	ISE	IAE	ITAE
6	1208	392.3	4903	616.4	274.7	3005
8	1004	352.4	4526	437.3	226.7	2509
10	205.0	155.9	1583.4	115	116.6	1119.6
12	89.77	102.1	974.97	72.8	90.9	866.8
14	56.47	80.88	769.51	45.6	71.3	674.1
16	36.28	64.36	610.86	28.9	56.3	528.4
18	23.76	51.54	485.19	18.6	45.2	419.4
20	16.14	41.75	386.45	12.6	37	337
22	11.36	34.65	310.04	8.9	30.8	273.8
24	8.54	29.25	249.67	6.8	26.3	227.7
26	6.87	25.15	202.46	5.6	23.1	195.6
28	5.9	22.2	166.38	4.9	21	172.8
30	5.38	20.12	139.27	4.5	19.6	157.8

5. Conclusions

We observed and quantified using performance criteria such as ISE, IAE, and ITAE that in systems without uncertainties (ideal systems) is a better choice to select a type-1 FLC since it works a little better than a type-2 FLC, and it is easier to implement it. It is known that type-1 FLC can handle nonlinearities, and uncertainties up to some extent.

Unfortunately, real systems are inherently noisy and nonlinear, since any element in the system contributes with deviations of the expected measures because of thermal noise, electromagnetic interference, etc., moreover, they add nonlinearities from element to element in the system.

In the simulation of real systems, systems with uncertainty, we observed that the results presented in Table III demonstrated that the performance of this kind of controllers is better under high noise levels. After optimizing the interval type- 2 MFs the performance of the system is improved as we can see in table V.

We can say that using a type-2 FLC in real world applications could be a better choice since this type of system is a more suitable system to manage uncertainty, as we can see in the results shown in tables IV and V.

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