Quality of Service Evaluation of Error Control for TCP/IP-Based Systems in Packet Switching ATM Networks

Khalid Darabkh and Ramazan Aygün, *Member, IEEE*

***ABSTRACT*—**Convolutional decoding using Fano algorithms is one of the most important techniques of channel coding used for error control. It is of interest since they have variable decoding time and are highly sensitive to the channel parameters such as an inaccurate estimate of channel SNR and an incomplete compensation of phase noise. The operation of Fano decoding in this paper is considered in TCP/IP-based ATM packet (fixed size packets) switching networks. The handshaking mechanism with packet-to-packet acknowledgments is implemented. In this paper, we complete a work has been done to compute numerically the mean buffer size by showing an analytical way to reach the final stage of providing explicit expressions for that performance metric. Analytical results are presented for several different channel conditions, packet arrival rates, and decoding time-out limits.

KEYWORDS—Convolutional code, Fano decoder, time-out limit, noisy channel, mean buffer size, queuing analysis.

I. INTRODUCTION

Channel coding is a way of adding redundant bits to the original data bits to guard the system against noise. Convolutional code is one of the most important techniques used for channel coding that its main goal to reduce the probability of erroneous transmission over noisy channel. Mainly there are two important decoding algorithms for convolutional codes, the Maximum-Likehood Decoding (Viterbi's algorithm), and Fano decoding.

Fano decoding algorithms [3] [8] are of interest since they have variable decoding time and it's computational and storage requirements grow linearly as a function of number of stages ,L, in the shift register that is used at the coder side. On the other hand the Maximum-likehood decoding (Viterbi's algorithm) [8] [9] has a fixed decoding time and its computational and storage requirements grow exponentially as a function of (L).

It has been shown both analytically and experimentally [2] [3] [4] [6] that the tail of the distribution of the Fano decoding time behaves according to the Pareto law. It is also known that the decoding time is also a function of the channel SNR (signal-to-noise ratio) [2] [5] [7].When the SNR is low, it is means the channel is noisy. Thus, the decoding time needs to be large, on the other hand, if the SNR is low, it means the channel is good (not noisy). Therefore, the decoding time needs to be small, the decoder is not allowed to run without limit in decoding time. Hence a timeout is often imposed to limit the decoding time of the Fano decoder. Figure 1 shows the Fano decoding system. We assume that packets from the channel arrive to an infinite buffer for the decoder according to a Bernoulli process, and that the decoding time of the Fano decoder follows the Pareto distribution parameterized by the channel SNR. In this paper, we complete a work has been done to compute numerically [2] the mean buffer size by showing an analytical way to reach the final stage of providing explicit expressions for that performance metric. We show that this work can be used in any application that support TCP/IP-based ATM packet (fixed size packets) switching networks where packet-to-packet acknowledgments is the handshaking mechanism that is implemented. Analytical results are presented for several different channel conditions, packet arrival rates, and decoding time out limits.

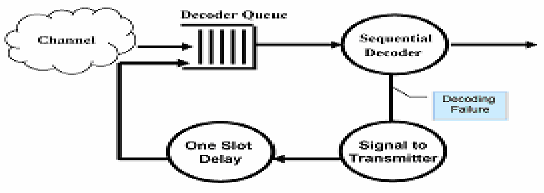

Figure 1. Fano decoder with an infinite buffer.

II. DISCRETE TIME MARKOV MODEL

The Fano decoding system is analyzed using discrete-time Markov model. This model is used for TCP/IP-based systems where packet-to-packet acknowledgment model is implemented. The time axis is portioned into slots of equal length where each slot corresponds to exactly the time to transmit a packet over the channel (propagation time + transmission time). We assume that all the incoming packets are assumed to be equally length. This is the case if we send Internet packets (typically of the size of 1Kbytes) over a wireless link (where packets have the size of around 300 bytes) or over the so called ATM networks (in which cells have the size of 52 bytes. Thus, the decoder can receive at most one new packet during a slot. The new packets arrive at the decoder from the channel according to Bernoulli process. A slot carries an arriving packet with probability λ and it is idle

---

Khalid Darabkh is with Computer Engineering Department, University of Alabama in Huntsville, Huntsville, AL 35899, USA (e-mail: darabkk @eng.uah.edu).

Ramazan Aygün is with Computer Science Department, University of Alabama in Huntsville, Huntsville, 35899, USA (e-mail: raygun @cs.uah.edu).

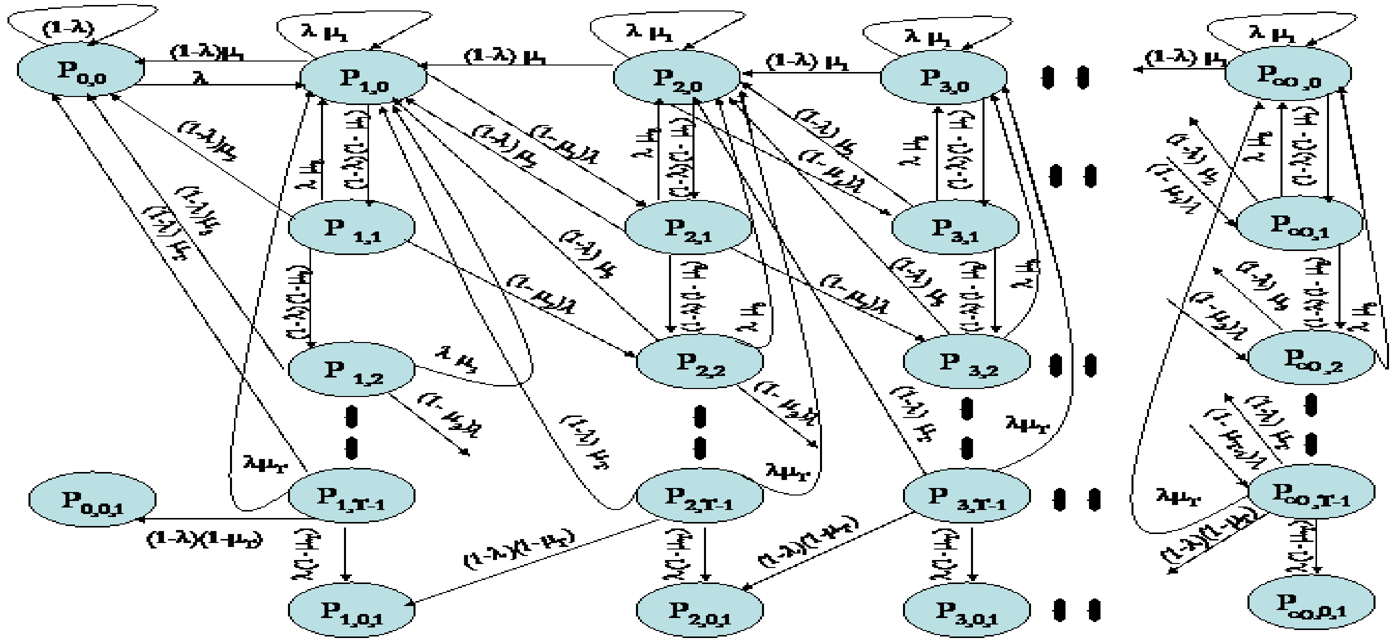

Figure 2: Probability state transitions for Geometric/Pareto/1 queue

(no transmission) with probability 1- λ. The service time follows the Pareto distribution with a parameter β which is a function of SNR. The buffer is assumed to be at least one room. We make another assumption that the decoding time of a packet is in chunks of length equal to the slot size. That is, the decoder can start and stop decoding only at the end of a slot.

We also assume in our simulation and analytical studies the same assumption used in [2] that any packet is allowed up to T slots for decoding. If a packet requires j slots for decoding ($j \leq T$), it leaves the system at the end of the j^th slot after the beginning of its decoding, and the decoding of a new packet starts (if there is a new packet in the decoder's buffer) at the beginning of the following slot. If a packet's decoding needs more than T slots, the decoder stops that packet's decoding after T slots. This packet cannot be decoded and thus a decoding failure results. Therefore the decoder signals packet's transmitter that a decoding failure occurs. Consequently, this packet is retransmitted at the following slot, while the decoder starts at that slot decoding of another packet if there is any in the buffer.

III. THEORETICAL ANALYSIS AND RESULTS

We use the notations that are mentioned in prior research in [2] [7]: c_k denotes the probability of decoding being completed in exactly k slots and μ_k denotes the conditional probability that decoding is completed in k slots given that the decoding is longer than (k – 1) slots. Then the conditional probability μ_k is given by $\mu_j = \frac{c_j}{1-F_{j-1}}$ where $F_j = \sum_{i=1}^{j} c_i$ is the cumulative distribution function (CDF) of the decoding time.

Let $P_{n,t,w}$ be the probability that the decoder's buffer contains n packets, including the one being decoded by the decoder, which is in the t^th slot of decoding and w packets that need to be retransmitted. Figure 2 shows the probability state transitions for $\mathrm{Geometric/Pareto/1/\infty}$ queue model. The summation of all the outgoing links (probabilities) from each state must be equal to one. For details about steady-state transitions equations [2] and distribution of the decoding time [7]: The following probability generating functions are defined:

$$P_j(z) = \sum_{n=1}^{\infty} p_{n,j} z^n, \quad 0 \leq j \leq T-1 \tag{1}$$

$$P_{0,1}(z) = \sum_{n=1}^{\infty} p_{n,0,1} z^n, \tag{2}$$

$$P(z) = p_{0,0} + p_{0,0,1} + p_{0,1}(z) + p_0(z) + \sum_{j=1}^{T-1} P_j(z). \tag{3}$$

$$P_0(z) = \frac{h_2(z)p_{0,0} + h_3(z)p_{0,0,1}}{h_1(z)} \tag{4}$$

$$h_1(z) = \left(z - z(1-F_T)f(z)^{T-1}\right)\left(z - \sum_{j=1}^{T} c_j f(z)^j\right) \tag{5}$$

$$W = \left((1-F_T)f(z)^T\right)\left(z\sum_{j=1}^{T} c_j f(z)^{j-1}\right) \tag{6}$$

$$h_2(z) = \lambda z(z-1)\left(z - z(1-F_T)f(z)^{T-1}\right) \tag{7}$$

$$h_3(z) = z\left(\left(z^2 - z^2(1-F_T)f(z)^{T-1}\right) - z\sum_{j=1}^{T} c_j f(z)^{j-1}\right) \tag{8}$$

$$P_{0,1}(z) = \frac{(1-F_T)f(z)^T}{\left(z - z(1-F_T)f(z)^{T-1}\right)} P_0(z) - \left[\frac{z}{\left(z - z(1-F_T)f(z)^{T-1}\right)} p_{0,0,1}\right] \tag{9}$$

$$P_j(z) = (1-F_j)F(z)^{j-1}\left(f(z)P_0(z) + zP_{0,1}(z)\right) \tag{10}$$

$$P_{0,0,1} = yP_{0,0} = \frac{\lambda(1-F_T)(1-\lambda)^T}{\sum_{j=1}^{T}(1-\lambda)^j c_j} P_{0,0} \tag{11}$$

For details how to find equations (4)-(11) see [2]

Using the fact that P(1) =1, we solve for $P_{0,0}$ and substitute that value in P(z)

$$P(1) = p_{0,0} + p_{0,0,1} + p_{0,1}(1) + \sum_{j=1}^{T-1} P_j(1) = 1 \tag{12}$$

After substituting the equations (5), (6), (7), and (8) on (4) at z = 1 we can get:

$$p_0(1) = \frac{\lambda(1-1)(F_T)p_{0,0} + (F_T - F_T)p_{0,0,1}}{F_T(1-F_T) - (1-F_T)F_T} = \frac{0}{0} \tag{13}$$

Since the output of equation (13) is 0/0, we take the limit:

$$P_0(1) = \lim_{z\to 1} \frac{h_2(z)p_{0,0} + h_3(z)p_{0,0,1}}{h_1(z)} \tag{14}$$

By using Hopital's law on equation (14) we can finally get:

$$p_0(1) = \frac{\left[F_T\lambda + y\left(\lambda\left[1 - T(1-F_T)\right] + F_T - \rho\right)\right] p_{0,0}}{(1-F_T)\lambda(1-T) - \rho + F_T} \tag{15}$$

After substituting z =1 on equation (9) we have:

$$p_{0,1}(1) = \frac{(1-F_T)}{F_T} p_0(1) - \frac{yp_{0,0}}{F_T} = \frac{p_0(1)}{F_T} - p_0(1) - \frac{yp_{0,0}}{F_T} \tag{16}$$

After substituting z =1 on equation (10) we can get:

$$P_j(1) = (1-F_j)\left(P_0(1) + P_{0,1}(1)\right) \tag{17}$$

Finally after substituting the equations (11), (15), (16), and (17) on (12) we can get:

$$P_{0,0} = \frac{F_T}{F_T(1+y) + (m-y)\left(\sum_{j=1}^{T-1}(1-F_j) + 1\right)} \tag{18}$$

Where

$$m = \frac{F_T\lambda + y\left(\lambda\left[1 - T(1-F_T)\right] + F_T - \rho\right)}{(1-F_T)\lambda(1-T) - \rho + F_T} \tag{19}$$

$$\rho = \lambda E(c) \tag{20}$$

ρ is the load of the infinite queueing system with E(c) being the expected decoding time.

$$E(c) = \sum_{j=0}^{T} jc_j \tag{21}$$

Now the average number of packets in the buffer is the derivative of P(z) at z = 1 [1]. Therefore, from (3)

$$P'(z)\Big|_{z=1} = P'_{0,1}(1) + \lambda\left(P_{0,1}(1) + P_0(1)\right)\sum_{j=1}^{T-1}(j-1)(1-F_j) + W_1 \tag{22}$$

where,

$$W_1 = \sum_{j=1}^{T-1}(1-F_j)\left(P'_{0,1}(1) + P'_0(1) + P_{0,1}(1) + \lambda P_0(1)\right) + P'_0(1) \tag{23}$$

By taking the derivative of equation (4) and applying the limit since the output is 0/0:

$$P'_0(z)\Big|_{z=1} = \lim_{z\to 1} \frac{h_1(z)\left(h'_2(z)p_{0,0} + h'_3(z)p_{0,0,1}\right) - \left(h_2(z)p_{0,0} + h_3(z)p_{0,0,1}\right)h'_1(z)}{\left(h_1(z)\right)^2} \tag{24}$$

By using Hopital's law on equation (24) we have:

$$P'_0(z)\Big|_{z=1} = \frac{h_1(z)\left(h''_2(z)p_{0,0} + h''_3(z)p_{0,0,1}\right) + h'_1(z)\left(h'_2(z)p_{0,0} + h'_3(z)p_{0,0,1}\right) - W_2}{2\left(h_1(z)\right)h'_1(z)} \tag{25}$$

$$W_2 = \left(h_2(z)p_{0,0} + h_3(z)p_{0,0,1}\right)h''_1(z) - \left(h'_2(z)p_{0,0} + h'_3(z)p_{0,0,1}\right)h'_1(z) \tag{26}$$

By taking the limit another time since still the result (0/0):

$$P'_0(z)\Big|_{z=1} = \lim_{z\to 1} \frac{h_1(z)\left(h''_2(z)p_{0,0} + h''_3(z)p_{0,0,1}\right) - \left(h_2(z)p_{0,0} + h_3(z)p_{0,0,1}\right)h''_1(z))}{2\left(h_1(z)\right)h'_1(z)} \tag{27}$$

By applying Hopital's law on equation (27) we can finally get:

$$P'_0(z)\Big|_{z=1} = \frac{h_1(1)\left(h'''_2(1)p_{0,0} + h'''_3(1)p_{0,0,1}\right) + \left(h''_2(1)p_{0,0} + h''_3(1)p_{0,0,1}\right)h'_1(1) - W_3}{2\left(h_1(1)\right)h''_1(1) + 2\left(h'_1(1)\right)^2} \tag{28}$$

where,

$$W_3 = \left(h'_2(1)p_{0,0} + h'_3(1)p_{0,0,1}\right)h''_1(1) - \left(h_2(1)p_{0,0} + h_3(1)p_{0,0,1}\right)h'''_1(1) \tag{29}$$

After substituting z =1 on the equations (5),(6),(7), and (8):

$$h_1(1) = 0\,,\ h_2(1) = 0\,,\ \text{and}\ \ h_3(1) = 0 \tag{30}$$

After substituting the results of (30) on (28) and (29), we can get:

$$P'_0(z)\Big|_{z=1} = \frac{\left(h''_2(z)p_{0,0} + h''_3(z)p_{0,0,1}\right)h'_1(z) - \left(h'_2(z)p_{0,0} + h'_3(z)p_{0,0,1}\right)h''_1(z)}{2\left(h'_1(z)\right)^2} \tag{31}$$

By taking the first derivative of equations (5), (6), (7), and (8) and substituting z =1, we can get:

$$h'_1(1) = F_T K_3 + (1-F_T)K_1 - \left((1-F_T)K_5 + F_T K_7\right) \tag{32}$$

$$h'_2(1) = \lambda F_T \tag{33}$$

$$h'_3(1) = K_9 - K_5 \tag{34}$$

By taking the second derivative of equations (5), (6), (7), and (8) and substituting z =1, we can get:

$$\begin{aligned} h''_1(1) = K_1K_3 - \left[(1-F_T)K_6 + K_5K_7 + F_TK_8 + K_7K_5\right] \\ + F_TK_4 + K_3K_1 + (1-F_T)K_2 \end{aligned} \tag{35}$$

$$h''_2(1) = 2\lambda F_T + 2K_1\lambda \tag{36}$$

$$h''_3(1) = K_{10} - K_6 + 2(K_9 - K_5) \tag{37}$$

where, $K_1 = 1 - (1-F_T)\left(\lambda(T-1) + 1\right)$ (38)

$$K_2 = -(1-F_T)(T-1)\lambda\left(\lambda(T-2) + 2\right) \tag{39}$$

$$K_3 = 1 - \rho \tag{40}$$

$$K_4 = -\lambda\left(\lambda E(d) - \rho\right) \tag{41}$$

$$K_5 = \rho + F_T(1-\lambda) \tag{42}$$

$$K_6 = \lambda\left[\lambda E(d) - 3\rho + 2\lambda F_T\right] + 2\left[\rho - \lambda F_T\right] \tag{43}$$

$$K_7 = (1-F_T)\lambda T \tag{44}$$

$$K_8 = T(T-1)\lambda^2(1-F_T) \tag{45}$$

$$K_9 = 2-(1-F_T)(\lambda(T-1)+2) \quad (46)$$

$$K_{10} = 2-(1-F_T)\left(\lambda^2(T-1)(T-2)+4\lambda(T-1)+2\right) \quad (47)$$

$$E(d) = \sum_{j=0}^{T} j^2 c_j \quad (48)$$

Now by taking the first derivative and substituting z =1 of equation (9), we can finally get:

$$P'_{0,1}(1) = \frac{(1-F_T)\left(F_T P'_0(1) + P_0(1)[F_T\lambda T - K_1]\right) - (F_T - K_1)p_{0,0,1}}{(F_T)^2} \quad (49)$$

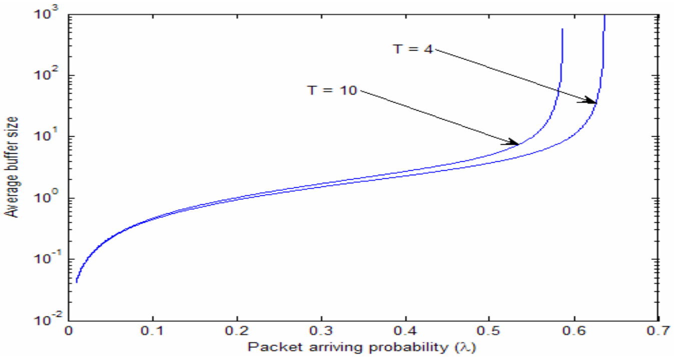

Figure 3. Average buffer occupancy versus packet arrival probability λ ($\beta = 1.5$)

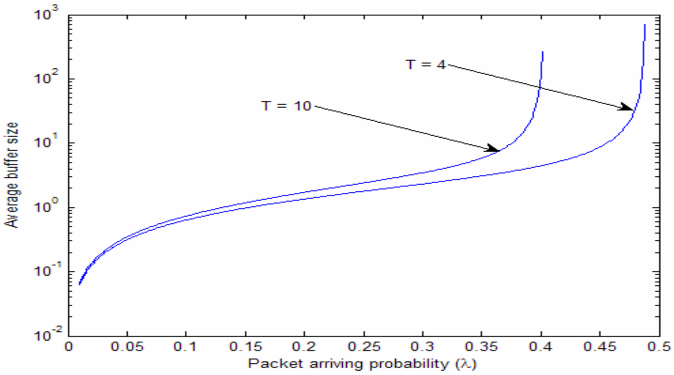

Figure 4. Average buffer occupancy versus packet arrival probability λ ($\beta = 1$)

It can be observed from Figures 3 and 4 for a fixed channel condition (β) and packet arriving probability, the increase in the average buffer occupancy as the decoding time-out limit (T) increases. A larger time-out limit may cause buffer overflow. It can be seen from Figures 3 and 4, for a fixed packet arriving probability and decoding time-out limit, the increase in the average buffer occupancy as β decreases. This is expected since a lower β means the channel is much noisy (more decoding time is required). It is also interesting to see from these Figures that the buffer capacity is so large (i.e. it can go to infinity) after reaching a certain limit of packet arrival probability. This is due to the constraint that set by the decoder queue on the packet rate which is the queue is stable for packet rate, for which $\text{packet rate} \le \frac{1}{E(c)}$.

IV. CONCLUSION

In this paper, we extend a work has been done to compute numerically the average buffer size of the Fano decoding system by providing analytical way to reach the final stage of providing explicit expression for that performance metric. The Fano decoding system is analyzed using discrete-time Markov model. We show that this model can be applied for TCP/IP systems that based on packet-to-packet acknowledgment mechanism. In this model, the time axis is portioned into slots of equal length where each slot corresponds to exactly the time to transmit a packet over the channel (propagation time + transmission time). All the incoming packets are assumed to be equally length. This is the case if we send Internet packets (typically of the size of 1Kbytes) over the so called ATM networks (in which cells have the size of 52 bytes. If a packet's decoding needs more than the time-out limit, this packet cannot be decoded and thus a decoding failure results. Therefore the decoder signals packet's transmitter that a decoding failure occurs. Consequently, this packet is retransmitted at the following slot, while the decoder starts at that slot decoding of another packet if there is any in the buffer. Analytical results are presented and explained for different channel conditions, packet arrival rates, and decoding time-out limits.

V. REFERENCES

[1] D. Cross, and C.M. Harris, "Fundamentals of Queuing theory", John Wily and Sons, 1998.

[2] N. Shacham, "ARQ with Sequential Decoding of Packetized Data," IEEE Trans. Commun., vol. COM-32, pp. 1118-1127, Oct. 1984.

[3] T. Hashimoto, "Bounds on a probability for the heavy tailed distribution and the probability of deficient decoding in sequential decoding", IEEE Trans. on Inform. Theory, vol. 51, issue 3, pp. 990 – 1002, March 2005.

[4] S. Kallel, and D. Haccoun, "Sequential decoding with an efficient partial retransmission ARQ strategy", IEEE Trans. on Commun. vol. 39, Issue 2, pp. 208 – 213, Feb. 1991.

[5] K. Darabkh, and W.D Pan, "Queueing Simulation for Fano Decoders with Finite Buffer Capacity", Accepted for 9th Communications and Networking Simulation Symposium conference (CNS'06), Huntsville, AL, April 2006.

[6] S. Kallel, and D. Haccoun, "Sequential decoding with ARQ and code combining: A robust-hybrid FEC/ARQ system," *IEEE Trans. Commun.*, vol. 36, pp. 773-780, July 1988.

[7] K. Darabkh, and W.D Pan, "Stationary Queue-Size Distribution for Variable Complexity Sequential Decoders with Large Timeout", Accepted for 44th ACM Southeast Conference, Melbourne, Florida, March 2006.

[8] Y. S. Han, P.-N. Chen, and H.-B. Wu, "A maximum-likelihood soft-decision sequential decoding algorithm for binary convolutional codes", IEEE Trans. Commun., vol.50, no.2, pp.173-178, February 2002.

[9] R. Johannesson and K. Sh. Zigangirov, "Fundamentals of Convolutional Coding", IEEE Press: Piscataway, NJ, 1999.