Performance degradation of OFDM and MC-CDMA to carrier phase jitter

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***Abstract*— This paper analyses the effect of carrier phase jitter on the performance of OFDM and MC-CDMA waveforms. Assuming a multicarrier downlink through an AWGN channel, we investigate the impact of jitter phase models on the SNR degradation. To this aim, we propose to consider a small random process, a gaussian process then a rayleigh process to model this jitter. Different performance degradation expressions, due to this default, are derived for these systems. Simulation results show better performance for a rayleigh model and same sensitivity for all waveforms studied.**

***Index Terms*— OFDM, MC-CDMA, phase jitter, modeling, performance degradation.**

I. INTRODUCTION

The radio environment is harsh, due to the many reflected waves and other effects. Using adaptative equalization techniques at the receiver could be the solution, but there are practical difficulties in operating this equalization in real time at several megabits per second with compact, low cost hardware. A promising candidate that eliminates a need for the complex equalizers is the Orthogonal Frequency Division Multiplexing (OFDM), a multiple carrier modulation technique [8]. This technique has recently gained a lot of attention and has been proposed for a large number of applications such as transmission over twisted pair cables and mobile radio.

A variation of OFDM is MultiCarrier CDMA (MC-CDMA) which is an OFDM technique where the individual data symbols are spread using a spreading code in frequency domain. The spreading code associated with MC-CDMA provides multiple access scheme as well as interference suppression. In fact these multicarrier systems offer immunity to channel dispersion, high bandwidth efficiency and high data rate [4]. Nevertheless, multicarrier systems are known to be sensitive to carrier phase error between carrier oscillator at the transmitter and the receiver. Many papers treats about the influence of such impairments.

All previous works assume that phase jitter is modeled by a small random stationnary process without any specification on the law followed by this jitter [2], [5]. Under this assumption, they point out that performance degradation of multicarrier systems depends only on the jitter variance. Our paper will complete these previous studies and, using specific probability models such as gaussian and rayleigh process, we will analyse the immunity of OFDM and MC-CDMA systems to phase jitter defaults. We will derive SNR (Signal to Noise Ratio) expressions and we will give simulation results.

II. SYSTEMS DESCRIPTION

A. OFDM system

The conceptual bloc diagram of an OFDM system is shown in Fig.1. In OFDM, the available bandwidth is split into a set of N_F orthogonal subchannels [1]. A sequence of complex data symbols is split into frames of N_F symbols $\{a_{i,m}; m = 0, \ldots, N_F - 1\}$, $a_{i,m}$ denoting the mth symbol transmitted during the ith symbol interval. The symbols $a_{i,m}$ modulate the orthogonal carriers, resulting in the time domain samples $s_{i,k}$ given by :

$$s_{i,k} = \sum_{m=0}^{N_F-1} \sqrt{E_{s,m}} a_{i,m} e^{j2\pi \frac{km}{N_F}} \tag{1}$$

where $E_{s,m}$ is the energy per symbol belonging to user m. Intersymbol interference, caused by the presence of a dispersive channel, can be avoided by cyclically extending the transmitted signal with a guard interval of length ν [1]. The extended samples are fed to transmit filter $p(t)$, a unit energy square root Nyquist filter. The resulting time domain signal to be sent over the channel is :

$$s(t) = \sum_{i=-\infty}^{+\infty} \sum_{k=-\nu}^{N_F-1} s_{i,k} p(t - (k + i(N_F + \nu))T) \tag{2}$$

where T is the symbol duration.

In the ideal channel, the transmitted signal is only disturbed by phase error $\phi(t)$ and by additive white gaussian noise $w_{i,k}$. This phase error is due to the divergence of local oscillator and the oscillator used for upconversion [7]. Then the received signal is fed to a filter matched to the transmit filter, sampled and keeping outside the guard interval for further processing. The resulting samples can be written as :

$$r_{i,k} = \sum_{m=0}^{N_F-1} \sqrt{E_{s,m}} a_{i,m} e^{j2\pi \frac{km}{N_F}} e^{j\phi(t)} + w_{i,k} \tag{3}$$

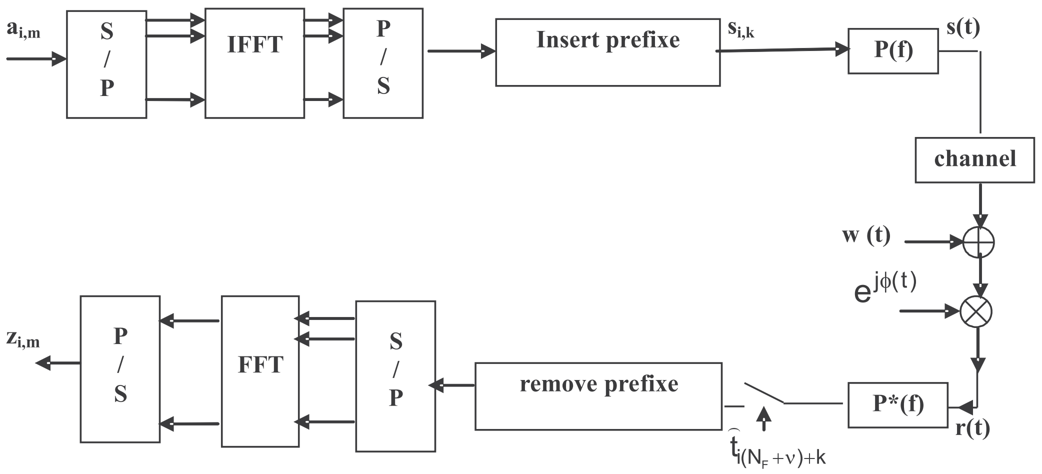

Fig. 1. Conceptuel block diagram of OFDM system.

The demodulation is done using a fast Fourier transform (FFT) to give the samples :

$$z_{i,m} = \frac{1}{N_F}\sum_{l=0}^{N_F-1}\sum_{k=0}^{N_F-1}\sqrt{E_{s,l}}\,a_{i,l}e^{j2\pi\frac{k(m-l)}{N_F}}e^{j\phi(t)} + w_{i,m} \tag{4}$$

where $w_{i,m}$ is the contribution of additive white noise.
By taking into consideration different interference terms, equation (4) can be simplified as follows :

$$z_{i,m} = \sum_{l=0}^{N_F-1}\sqrt{E_{s,l}}\,a_{i,l}I_{i,l,m} + w_{i,m} \tag{5}$$

where

$$I_{i,l,m} = \frac{1}{N_F}\sum_{k=0}^{N_F-1}e^{j2\pi\frac{k(m-l)}{N_F}}e^{j\phi(t)} \tag{6}$$

In the sum described in equation (5), we can distinguish the case of $l = m$ and the case of $l \neq m$, yielding the samples:

$$z_{i,m} = \sqrt{E_{s,m}}\,a_{i,m}I_{i,m,m} + \sum_{l=0\ et\ l\neq m}^{N_F-1}\sqrt{E_{s,l}}\,a_{i,l}I_{i,l,m} + w_{i,m} \tag{7}$$

The first term in equation (7) is referred to the useful component. This contribution can be further decomposed into an average useful component $E(I_{i,m,m})$ and a zero mean fluctuation $I_{i,m,m} - E(I_{i,m,m})$ or self interference (SI). The second contribution $(l \neq m)$ is the intersymbol interference (ISI), caused by other symbols. The last contribution is the additive noise term of average N_0.

B. MC-CDMA system

The conceptual bloc diagram of a downlink MC-CDMA system is shown in Fig.2. The data symbols $a_{i,l}$ transmitted to user l during the ith symbol interval are multiplied with corresponding chip $c_{iN_c+n,l}$, where n is the chip index and N_c is the spreading factor. In fact, in a multiuser scenario, each user is assigned a unique spreading sequence. The sequences are considered orthogonal and consisting of user-dependent Walh-Hadamard sequences multiplied with a complex value random scrambling sequence that is common to all active users [3], [6]. The resulting samples are modulated on the orthogonal carriers (OFDM) using an IFFT of length N_F. To avoid intersymbol interference, a guard interval of ν samples can be introduced. The obtain signal is, then, fed to the transmit pulse $p(t)$. The resulting signal to be sent over the channel is given by :

$$s(t) = \sum_{l=1}^{N_u}\sum_{i=-\infty}^{+\infty}\sum_{k=-\nu}^{N_F-1}s_{i(N_F+\nu)+k,l}\,p(t-(i(N_F+\nu)+k)T-\tau_c) \tag{8}$$

where

$$s_{i,l} = \frac{1}{\sqrt{N_F N_c}}\sum_{k=-\nu}^{N_F-1}\sum_{n=0}^{N_c-1}a_{i,l}c_{iN_c+n,l}e^{j2\pi\frac{kn}{N_F}} \tag{9}$$

In expression (8), T is the symbol period and τ_c is the time delay corresponding to phase clock transmitter. The signal is disturbed by additive white gaussian noise and a carrier phase jitter, $\phi_l(t)$. This jitter, was first modeled as a zero mean random process with jitter variance $\sigma^2_{\phi_l}$ [2], [5]. This random jitter has an unknown behavior. The sensitivity of MC systems performance to this random jitter has been investigated in [3]. Here, we propose to model the carrier phase jitter by gaussian and rayleigh laws and to see the effect of such jitter on downlink MC-CDMA and uplink MC-CDMA systems performance.
At the receiver, the signal is applied to a filter matched to the transmit filter then sampled. The ν samples of guard interval are disregarded from the obtain samples. At the output of the FFT bloc used for demodulation, samples are multiplied with

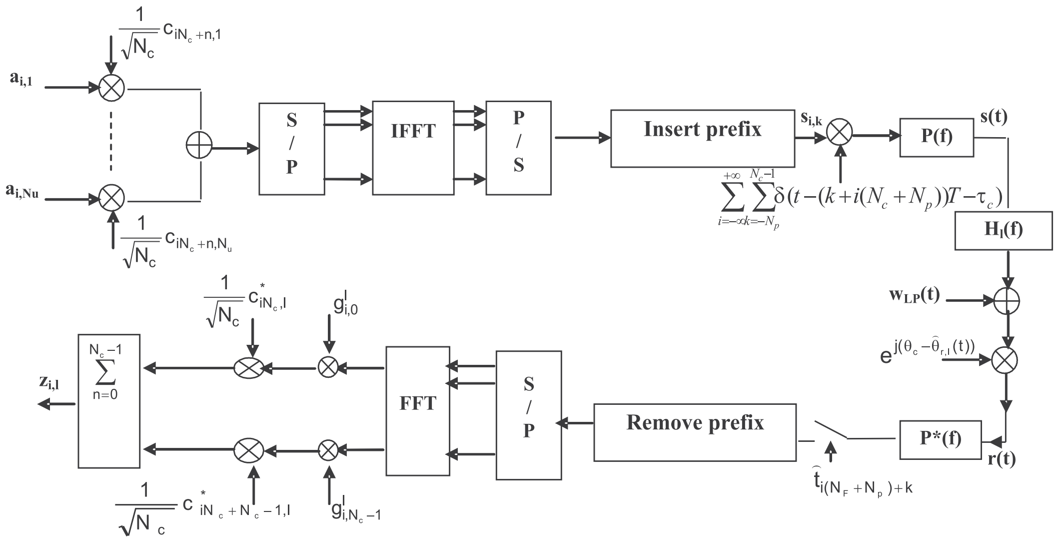

Fig. 2. Conceptuel block diagram of a downlink MC-CDMA system.

$g^{l}_{i,n}$ coefficients of the equalizer and with the chip $c^{*}_{i'N_c+n',l'}$ of considered user, then summed yielding:

$$
\begin{aligned}
z_{i,l} = \frac{\sqrt{N_F}}{\sqrt{N_F+N_P}} & \sum_{i'=-\infty}^{+\infty}\sum_{l'=1}^{N_u}\delta_{i,i'}a_{i',l'} \\
& \times\left[\frac{1}{N_c}\sum_{n,n'=0}^{N_c-1}c^{*}_{iN_c+n,l}c_{i'N_c+n',l'}\right] \\
\times\, g^{l}_{i,n} & \left[\frac{1}{N_F}\sum_{k=0}^{N_F-1}e^{-j2\pi\frac{k(n-n')}{N_F}}\right]e^{j\phi_l(t)}+w_{i,l} \qquad (10)
\end{aligned}
$$

where $w_{i,l}$ is the contribution of the additive white noise. We can simplify and rewrite expression (10) as follows :

$$
\begin{aligned}
z_{i,l} = \sqrt{\frac{N_F}{N_F+N_P}} & \Bigg[a_{i,l}I_{i,i,l,l}+\sum_{i'=-\infty\ et\ i'\neq i}^{+\infty}a_{i',l}I_{i,i',l,l} \\
& +\sum_{i'=-\infty}^{+\infty}\sum_{l'=0\ et\ l'\neq l}^{N_u-1}a_{i',l'}I_{i,i',l,l'}+w_{i,l} \qquad (11)
\end{aligned}
$$

where

$$
\begin{aligned}
I_{i,i',l,l'} = \frac{1}{N_cN_F}\delta_{i,i'} & \sum_{n,n'=0}^{N_c-1}c^{*}_{iN_c+n,l}c_{i'N_c+n',l'}g^{l}_{i,n} \\
& \times\sum_{k=0}^{N_F-1}e^{-j2\pi\frac{k(n-n')}{N_F}}e^{j\phi_l(t)} \qquad (12)
\end{aligned}
$$

The quantity $I_{i,i',l,l'}$ represents the contribution of the symbol $a_{i',l'}$ to the output of the receiver of the l^{th} user during the i^{th} MC-CDMA block.

In equation (12), the first contribution is the useful component. This contribution can be further decomposed into an average useful component $E(I_{i,i,l,l})$ and a zero mean fluctuation $I_{i,i,l,l}-E(I_{i,i,l,l})$ or self interference (SI). The second contribution $(i'\neq i$ and $l'=l)$ is the intersymbol interference (ISI), caused by other symbols from the same user. The third contribution $(l'\neq l)$ denotes the multiuser interference (MUI). The last contribution is the additive noise term.

III. PERFORMANCE DEGRADATION

In this section, we compute the degradation (in decibels) of the signal to noise ratio (SNR) when carrier phase jitter is present for OFDM and MC-CDMA systems. The performance degradation of these systems is studied with small random process, gaussian process and rayleigh process for phase jitter ϕ. If SNR(0) is the signal to noise ratio in the absence of carrier phase jitter, then, the degradation is given by :

$$deg=-10\log\frac{SNR(\phi)}{SNR(0)} \qquad (13)$$

At this point, we introduce the term $E(e^{j\phi})$ as the average of the carrier phase jitter modeled by small random process, gaussian or rayleigh distributions.

For a jitter phase modeled by a small random process without any specification on the law followed by this jitter, we can write :

$$e^{j\phi(t)}=1+j\phi(t) \qquad (14)$$

If we consider zero-mean jitter, then the term $E(e^{j\phi})$ is simplified as :

$$E(e^{j\phi(t)})=1 \qquad (15)$$

But with the assumption of gaussian model, $E(e^{j\phi})$ is given by :

$$E(e^{j\phi}) = \int_0^{2\pi} \frac{1}{\sigma_\phi\sqrt{2\pi}} e^{-\frac{\phi^2}{2\sigma_\phi^2}+j\phi} d\phi \qquad (16)$$

Whereas, for a rayleigh model, the quantity $E(e^{j\phi})$ becomes:

$$E(e^{j\phi}) = \int_0^{2\pi} \frac{\phi}{\sigma_\phi^2} e^{-\frac{\phi^2}{2\sigma_\phi^2}+j\phi} d\phi \qquad (17)$$

Regarding expression (16) and expression (17), we can note that there is no classical method to calculate the expressions above. Hence, we can hardly found the exact value of these integrals. An approximate method of calculation is to be used, based on trapezoids surface approximation.

A. OFDM system

In the case of OFDM system, the signal to noise ratio can be defined as the ratio of the average useful component to the sum of the powers of the other contributions. This SNR is, then, given by :

$$SNR(\phi) = \frac{\frac{N_F}{N_F+\nu} E_{s,m} P_u}{N_0 + \frac{N_F}{N_F+\nu} E_{s,m}(P_{SI} + P_{ISI})} \qquad (18)$$

In the absence of synchronization error, the signal to noise ration can be reduced to :

$$SNR(0) = \frac{N_F}{N_F+\nu} \frac{E_{s,m}}{N_0} \qquad (19)$$

In order to determine the exact value of expression (13), we give the different power expressions by :
$P_u = E(|I_{i,m,m}|^2)$
$P_{SI} = E(|I_{i,m,m} - E(I_{i,m,m})|^2)$
$P_{ISI} = \sum_{i=-\infty}^{+\infty} E(|I_{i,l,m}|^2)$
For a jitter phase modeled by a small random process without any specification on the law followed by this jitter, the SNR degradation given in expression (13) becomes :

$$deg = 10 \log(1 + SNR(0)(\sigma_\phi^2 + 1)) \qquad (20)$$

If the jitter is modeled by gaussian or rayleigh models then, the performance degradation will be :

$$deg = -10 \log \frac{|A|^2}{1 + SNR(0)(2 - |A|^2)} \qquad (21)$$

Where $|A|$ is the module of the approximated value of $E(e^{j\phi})$ given by expression (16) in the case of gaussian phase jitter model and by expression (17) in the case of rayleigh phase jitter model.
From expression (21), the performance degradation of OFDM system depends not only on jitter variance but also on the law followed by this jitter.

B. MC-CDMA system

In the case of MC-CDMA system, the signal to noise ratio degradation can be defined as :

$$deg = -10 \log \frac{P_u}{1 + SNR(0)(P_{SI,l} + P_{ISI,l} + P_{IUI,l})} \qquad (22)$$

where

$$SNR(0) = \frac{N_F}{N_F+\nu} \frac{E_{s,l}}{E\,|w_{i,l}|^2} \qquad (23)$$

The powers of the useful component, the self interference, the intersymbol interference and the interuser interference, are given by :

$P_{u,l} = |E(I_{i,i,l,l})|^2$
$P_{SI,l} = E(|I_{i,i,l,l} - E(I_{i,i,l,l})|^2)$
$P_{ISI} = \sum_{i'=-\infty}^{+\infty} E(|I_{i,i',l,l}|^2)$
$P_{IUI} = \sum_{l'=0}^{N_u-1} \frac{E_{s,l'}}{E_{s,l}} E(|I_{i,i,l,l'}|^2)$

We easily note that $P_{ISI,l} = 0$ since the intersymbol interference $I_{i,i',l,l} = 0$ for $(i' \neq i)$.
Assuming that all users have the same jitter variance σ_ϕ^2 and the same energy per symbol E_s, and with highest load, i.e. $(N_u = N_c)$, we can simplify the different power expressions. Similarly to the previous subsection, the expression (22) becomes for a small random process:

$$deg = 10 \log(1 + SNR(0)(\sigma_\phi^2 + 1)) \qquad (24)$$

and for a gaussian or a rayleigh process

$$deg = -10 \log \frac{|A|^2}{1 + SNR(0)(2 - |A|^2)} \qquad (25)$$

Where $|A|$ is the module of the approximated value of $E(e^{j\phi})$ given by expression (16) in the case of gaussian phase jitter model and by expression (17) in the case of rayleigh phase jitter model.
From expressions (24) and (25), the performance degradation of MC-CDMA system depends on the jitter variance σ_ϕ^2 with number of active users equal to the number of carriers. And since the value of $|A|$ depends on the jitter model used, i.e. gaussian or rayleigh, then the degradation is independent of the number of carriers, the spreading factor and the jitter spectrum, but depends on jitter variance and probability law used for modeling carrier phase jitter.

IV. INTERPRETATIONS AND CONCLUSIONS

Looking at expressions (20), (21), (24) and (25) we remark that the SNR degradation has same sensitivity to phase jitter models for different waveforms OFDM and MC-CDMA. These waveforms perform better with a rayleigh process as shown in Fig.3. In this figure, we represent a comparison of performance degradation for OFDM and MC-CDMA systems in presence of phase jitter with SNR(0)=30dB and assuming small random, gaussian and rayleigh jitter models. We found that, with rayleigh model, we have minimum degradation. So by investigating the sensitivity of OFDM and MC-CDMA to

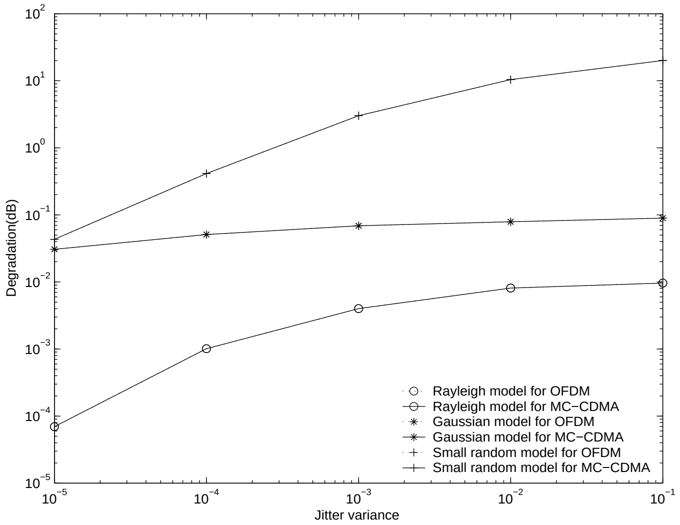

Fig. 3. Comparison of performance degradation for OFDM and MC-CDMA systems in presence of phase jitter with SNR(0)=30dB and assuming small random, gaussian and rayleigh jitter models

carrier phase errors, we point out that OFDM and MC-CDMA have essentially the same sensitivities to phase jitter. And, by comparing small random jitter, gaussian jitter and rayleigh jitter models, we conclude that these waveforms perform better with rayleigh model.

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