On the Use of Adaptive Loading algorithms in MIMO-OFDM Systems

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Abstract - *In this paper, Optimum adaptive loading algorithms are applied to Multiple-Input Multiple-Output Orthogonal Frequency Division Multiplexing (MIMO-OFDM) system. Chaw's and Compello's algorithms have been implemented to obtain a bit and power allocation for each subcarrier assuming instantaneous channel knowledge. Channel matrix singular value decomposition (SVD) method has been employed in MIMO-OFDM system in order to overcome subchannel interference. The effectiveness of these techniques has demonstrated through the simulation of an Adaptive MIMO-OFDM system, and the comparison shown that the proposed algorithms ensure better performance in the case of MIMO vs. SISO.*

Keywords: OFDM, SISO, MIMO, SVD, Adaptive Loading.

1 Introduction

Orthogonal frequency division multiplexing (OFDM) has become a popular technique for transmission of signals over wireless channels. OFDM has been adopted in several wireless standards such as digital audio broadcasting (DAB), digital video broadcasting (DVB-T), the IEEE 802.11a [1] local area network (LAN) standard and the IEEE 802.16a [2] metropolitan area network (MAN) standard. OFDM converts a frequency-selective channel into a parallel collection of frequency flat subchannels. The subcarriers have the minimum frequency separation required to maintain orthogonality of their corresponding time domain waveforms, yet the signal spectra corresponding to the different subcarriers overlap in frequency. Hence, the available bandwidth is used very efficiently. If knowledge of the channel is available at the transmitter, then the OFDM transmitter can adapt its signaling strategy to match the channel.

Adaptive modulation is an important technique that yields increased data rates over non-adaptive uncoded schemes [3]. An inherent assumption in channel adaptation is some form of channel knowledge at both the transmitter and the receiver. Given this knowledge, both the transmitter and receiver can have an agreement upon modulation scheme for increased performance.

One of the major limiting factors in wireless communications is the scarcity of the spectrum. MIMO is a new technology that can dramatically increase the spectral efficiency by using antenna arrays at both the transmitter and receiver. A MIMO system takes advantage of the spatial diversity that is obtained by spatially separated antennas in a dense multipath scattering environment.

This paper is organized as follows. In Section 2, the system model of MIMO/OFDM is introduced. Then, we consider the power optimization and bit allocation algorithm for MIMO/OFDM in section 3. In Section 4, we apply SVD techniques to MIMO channel. Section 5 presents some comparative simulation results and finally, we have some concluding remarks in Section 6.

2 MIMO-OFDM System Model

2.1 OFDM Details

The block diagram of OFDM system studied in this paper is shown in Figure 1. A serial-to-parallel buffer segments the information bit sequence into parallel output stream and then modulator blocks map them into complex numbers which determines the constellation points of each subcarrier. The number of bits assigned to each subcarrier is variable based on the variability of signal to noise ratio across the frequency range. Optimization of this bit assignment will be detailed in further sections.

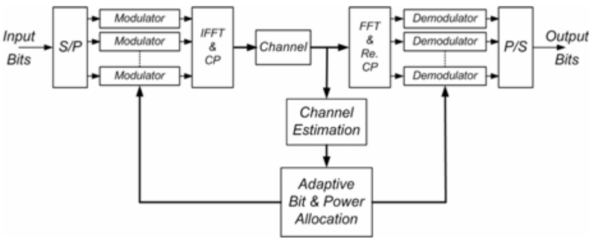

Figure 1: OFDM System block Diagram

The OFDM modulation can be efficiently implemented in discrete time using an inverse FFT (IFFT) to act as a modulator and an FFT to act as a demodulator. A cyclic prefix which is set to the excess delay of the radio channel is also added to each of the resulting signals to reduce the effect of ISI and inter-subcarrier interference. The sample streams are then converted from parallel-to-serial for final transmission.

2.2 MIMO Systems

MIMO systems can be defined simply. Given an arbitrary wireless communication system, we consider a link for which the transmitting ends as well as the receiving end is equipped with multiple antenna elements. Such a setup is illustrated in Fig. 2. The idea behind MIMO is that the signals on transmit (TX) antennas at one end and receive (RX) antennas at the other end are “combined” in such a way that the quality (bit-error rate or BER) or the data rate (bits/sec) of the communication for each MIMO user will be improved.

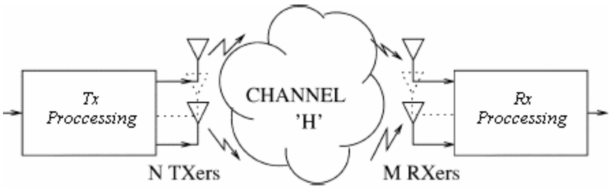

Figure 2: Diagram of a MIMO wireless transmission system.

3 Adaptive Modulation

The variation in fading statistics among different subcarriers in some OFDM channels suggests that some good subcarriers with high channel power gain can be made to carry more bits and/or be allocated with less transmission power, and vice versa for the weak subcarriers. The optimal adaptive transmission scheme, which achieves the Shannon capacity for a fixed transmit power, is the water filling distribution of power over the frequency selective channel [4]. However, while the water filling distribution will indeed yield the optimal solution, it is difficult to compute.

3.1 Loading Algorithms

There are two types of loading algorithms – those that try to maximize data rate and those that try to maximize performance at a given fixed data rate [3], [8].

Definition 1 (RATE-ADAPTIVE (RA) loading criterion) A rate-adaptive loading procedure maximizes (or approximately maximizes) the number of bits per symbol subject to a fixed energy constraint:

$$max\ b = \sum_{n=1}^{N} \frac{1}{2} log_2 \left(1 + \frac{\varepsilon_n \cdot g_n}{\Gamma} \right) \tag{1}$$

$$Subject\ to: N\,\bar{\varepsilon}_x = \sum_{n=1}^{N} \varepsilon_n \tag{2}$$

Definition 2 (MARGIN-ADAPTIVE (MA) loading criterion) A margin-adaptive loading procedure minimizes (or approximately minimizes) the energy subject to a fixed bits/symbol constraint:

$$min\ \varepsilon_x = \sum_{n=1}^{N} \varepsilon_n \tag{3}$$

$$Subject\ \ to:\ b = \sum_{n=1}^{N} \frac{1}{2} log_2 \left(1 + \frac{\varepsilon_n \cdot g_n}{\Gamma} \right) \tag{4}$$

The so-called “SNR gap”, denoted by Γ, is defined as a ratio of ideal SNR at which the system can transmit at C bits/symbol over a practical SNR at which the system can transmit R bits/symbol. It is a measure of how well the practical system compares to an ideal modulation system [3].

Loading algorithms compute values for b_n and ε_n for each and every subchannel in a parallel set of subchannels. One example of a loading algorithm is the optimum water-filling algorithm that solves a set of linear equations with boundary constraints. The solution of these water-filling equations for large N may produce b_n that have fractional parts or be very small. Such small or fractional b_n can complicate encoder and decoder implementation. Alternative suboptimal loading algorithms approximate the water-fill solution, but constrain b_n to integer values.

There are different algorithms proposed to implement the criterions express above, but in this paper we implement chow’s and Compello’s algorithms.

3.1.1 Chow’s Algorithm

Chow was able to verify that an “on/off” energy distribution, as long as it used the same or nearly the same transmission band as water-filling, exhibits negligible loss with respect to the exact water-filling shape[7]. To initialize the bit allocation, the scheme of [3], and [5] is employed. The procedure is summarized as follows (Algorithm Initialization):

1. Compute the subchannel signal to noise ratios.
2. Compute the number of bits for the ith subchannel based on the formula:

$$\hat{b}(i) = log_2 (1 + SNR(i)/\Gamma) \tag{5}$$

3. Round the value of $\hat{b}(i)$ down to $b(i)$.
4. Restrict $b(i)$ to take values 0, 1, 2, 4, 6 or 8.
5. Compute the energy for the ith subchannel based on the number of bits initially assigned to it using the formula:

(1)
(2)

$$e_i(b(i)) = \left(2^{b(i)} - 1\right) / \Gamma(i) \\ Where\quad GNR(i) = SNR(i)/\Gamma \tag{6}$$

3.1.2 Optimum Discrete Loading Algorithms

The basic concept of Optimum discrete loading algorithms is that each increment of additional information to be transported by a multi-channel transmission system is placed on the subchannel that would require the least incremental energy for its transport [3]. Levin and Campello have independently formalized an iterative algorithm that will translate any bit distribution into an efficient bit distribution. Efficiency means that there is no movement of a bit from one subchannel to another that reduces the symbol energy. Given the initial bit allocation, the B-tightness simply states the correct number of bits is being transmitted and optimized the bit allocation [3], [9]. A B-tightening (BT) algorithm is:

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1. Set $\hat{b} = \sum_{n=1}^{N} b_b$                                   (7)
2. While $\hat{b} \neq b$
   If $\hat{b} > b$
      (a) $n \leftarrow arg\{max_{1 \le i \le N}[e_i(b_i + \beta)]\}$
      (b) $\hat{b} \leftarrow \hat{b} - \beta$
      (c) $b_n \leftarrow b_n - \beta$
   Else
      (a) $m \leftarrow arg\{min_{1 \le i \le N}[e_i(b_i + \beta)]\}$
      (b) $\hat{b} \leftarrow \hat{b} + \beta$
      (c) $b_n \leftarrow b_n + \beta$
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An example of implementing these tow algorithms is shown in figure 3. This figure illustrates the typical channel frequency response, the discrete bit allocation to each tone, and the corresponding energy on each tone. As expected, the tones experiencing very poor channel instances had few or zero bits allocated to them. Also, it is interesting to note that the finite number of MQAM constellations available means that the rate remains fixed over some intervals where the gain does not vary too widely.

4 SVD of MIMO Subchannel

Channel matrix singular value decomposition (SVD) method is employed in MIMO-OFDM systems in order to overcome subchannel interference, to allocate transmitted Bit/Power through subchannels in an optimum manner. The SVD technique can be directly applied to the MIMO channel decomposition when perfect CSI knowledge at the transmit side is available. Applying a unitary pre-filtering and post-filtering (shaping) matrix to the transmitted and received signal on the n-th subcarrier, respectively, the MIMO channel can be turned into a couple of conventional SISO channels [10]. In the notation of matrices, the matrix H has singular value decomposition:

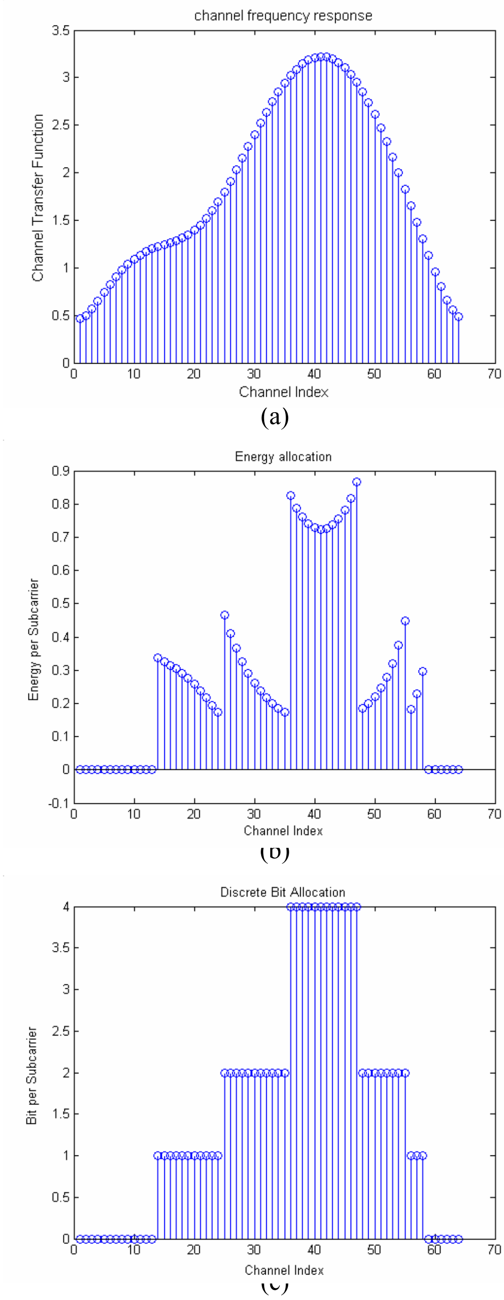
$$H_i = U_i \Lambda V_i^* \tag{8}$$

In this case we can define a received signal matrix as:

$$Y = HX + N = U^H HVX + U^H N \tag{9}$$

$$Y = U^H U \Lambda V^H VX + N_1 = \Lambda X + N_1 \tag{10}$$

Where U_i and V_i are unitary matrices, and Λ is the diagonal matrix of singular values of H_i. Note that the operator (.)* is the conjugate transpose operator.



(a)

(b)

(c)

Figure 3: Example optimal Bit and Energy Allocation

Now, if we use a transmit precoding filter of Vi and a receiver shaping filter of U_i, the equivalent MIMO channel between the IFFT and FFT blocks decomposes into parallel subchannels [11]. Therefore, we can use each parallel subchannels SNR to specify the number of bits and energy. Note that the number of such subchannels is exactly equal to the number of nonzero singular values of Hi. We apply same decomposition to each subchannel of the OFDM system. In general each precoder and shaping matrix will be different for different subchannels. Therefore, we can optimize the transmitter since at low and high SNRs the SVD method demonstrates good transmission performance.

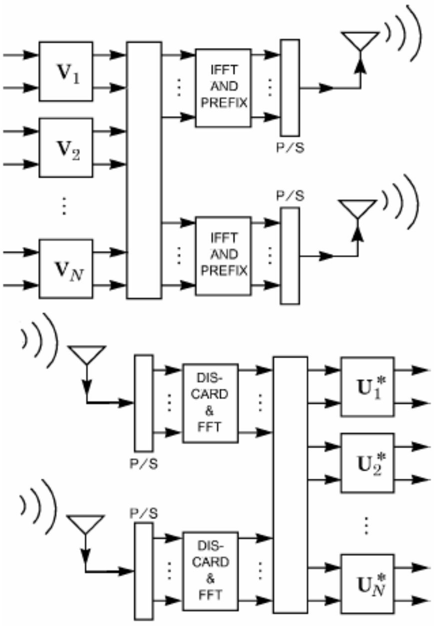


Figure 4: Using SVD in MIMO-OFDM System

5 Simulation and Results

In this section, the achievable Bite Error Rates of the following three cases are compared by Monte Carlo simulations: (1) Fixed-rate MIMO-OFDM System; (2) Adaptive SISO-OFDM System; and (3) Adaptive MIMO-OFDM System. Throughout the simulation, in order to simplify the model, the entire system is only considered as a discrete-time system. Perfect channel state information has been assumed. The configurations considered are for OFDM system with 256 subcarriers, 64 OFDM symbol time periods, and 16 symbol periods for guard time. The above parameters were held constant throughout the simulation. In the simulation, the channel of each link has 1×10-3 noise variance and exponential power delay profile.

In figure 5, we provide the simulation result for Adaptive SISO-OFDM, Fixed-rat and Adaptive MIMO-OFDM. In all simulations the MIMO system was held as a 2 × 2 links. Note that increased averaging (more Monte Carlo iterations) would surely smooth out the BER curves. In figure 6, we provide the simulation result for Adaptive MIMO-OFDM with different number of transmit and receive antennas.

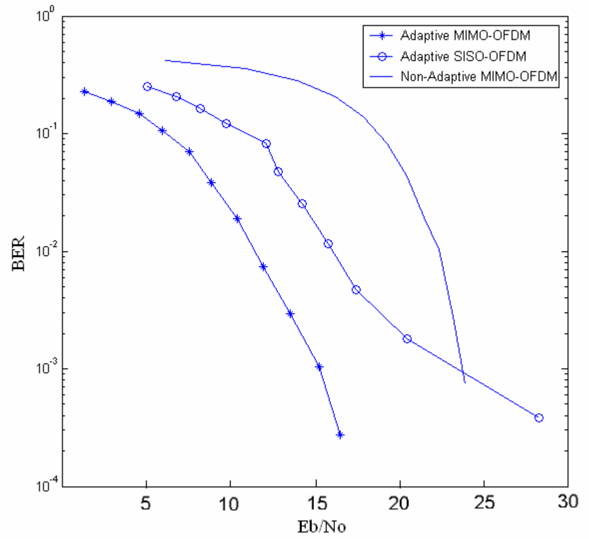


Figure 5: Comparison of adaptive loaded SISO-OFDM & MIMO-OFDM systems with fixed-rate MIMO-OFDM system

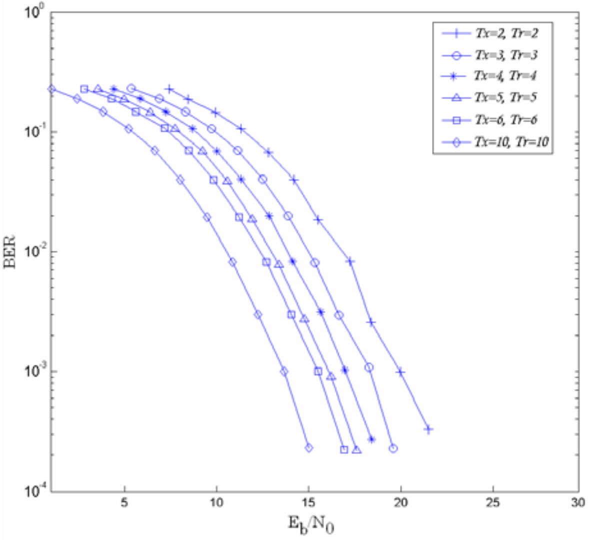


Figure 6: Comparison of adaptive loaded MIMO-OFDM systems with different numbers of transmit and receive antennas

6 Conclusions

Adaptive Loading algorithms and MIMO-OFDM systems are investigated in this paper. We introduce the optimum discrete bit loading algorithms and explore the manner of combining these algorithms with MIMO systems. We have found the necessity of using SVD method in MIMO-OFDM system to achieve the optimum bit loading. Simulation results show that at any given BER the adaptive SISO system will be outperformed by the adaptive MIMO system.

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8 References

[1] *Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications: High-Speed Physical Layer in the 5 GHz Band*, IEEE Standard 802.11a-1999.

[2] Local and Metropolitan Area Networks—Part 16, Air Interface for Fixed Broadband Wireless Access Systems, IEEE Standard IEEE 802.16a.

[3] J. Cioffi, *Digital Communications III*, EE379c, class notes. Stanford University, 2002.

[4] R. G. Gallager, *Information Theory and Reliable Communication*, Willey, New York, NY, 1968.

[5] D. Hughes-Hartogs. “Ensemble Modem Structure for Imperfect Transmission Media,” U.S. Patents Nos. 4,679,227 (July 1987), 4,731,816 (March 1988) and 4,833,706 (May 1989).

[6] Fischer, Robert EH. and Huber, Johannes B. “A New Loading Algorithm for Discrete Multitone Transmission,” Proceedings of GLOBECOM 96, pp. 724-728, London, UK 18-22 NOV. 1996.

[7] P. Chow, J. M. Cioffi, and J. A. C. Bingham, “A Practical Discrete Multitone Transceiver Loading Algorithm for Data Transmission Over Spectrally Shaped Channels,” *IEEE Trans. Comm*., Vol. 43, No. 2, February 1995, Page 773-775.

[8] J. Campello: “Optimal discrete bit loading for multicarrier modulation systems”, Proc. of the IEEE Int. Symp. On Information Theory, p. 193, Cambrige, MA, USA, Aug. 1998.

[9] J. Campello: “Practical bit loading for DMT”, IEEE International Conference on Communications ICC99, pp. 801-805, 1999.

[10] J. H. Sung and J. R. Barry, “Space-Time Processing with Channel Knowledge at the Transmitter”, European Conference on Communications, Bratislava, vol. 1, pp. 26-29, July 5-7, 2001

[11] A. Goldsmith, EE359 Wireless Communications. Course reader, Fall 2001.