

Heuristic Algorithm for Location-Allocation Problem Based on Wavelet Analysis in Integrated Logistics Distribution

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Abstract –A reliable model for Location-Allocation (LA) is proposed based on 0-1 mixed integer programming model presented by Baumol-Wolfe. A heuristic algorithm (HA) based on wavelet analysis is used to solve LA problem, which incorporates different customers' demands into a minimum wrap-net. It indicates that customers are allocated to one facility or one cluster if they satisfy that the distance is nearest to the facility. The model in the method involves the respective distance and demand of the different customers, which can solve real planar LA. A case study using computer simulation shows that the HA system achieves significant improvement compared to a recent LA solution.

Keywords: Location-Allocation (LA), wavelet analysis, heuristic search, minimum wrap-net method, logistics systematic optimization.

1 Introduction

All companies that aim to be competitive in market should pay attention to how their organizations deal with the entire supply chain. In particular, companies should focus on increasing the efficiency of their logistics operations. It is important for enterprises to develop optimal systems of logistics distribution under e-commerce through integrating their logistics activities. Location Allocation problem (LA) is an important branch of routing optimization in integrated logistics systems, which can be involved in all logistics distribution corporations.

During the last two decades or so, many scholars have developed more efficient problem solving techniques using the concept of integrated logistics systems, including optimizing models and applying associated algorithms for solving these problems. Vehicle Routing Problem (VRP) and location-allocation (LA) were developed and analyzed in logistics systematic optimization. Alan T. Murray and Ross A. Gerrard [1] introduced the capacitated regionally constrained p-

medium problem, which incorporates regional requirements in a LA framework, in addition to ensuring that maximum capacity limitations are maintained. Dijin Gong et al. [2] discussed an extension of the LA model, which has capacity constraints, and proposed an Evolutionary Strategy (ES) combined with efficient traditional optimization techniques. Some scholars used the Benegers' method that ensures the convergence for the model with the constraint equations [3-7]. This helps because the model and solution for LA are constructed in a way that provides for further LRP in integrated logistics. Location Routing Problem (LRP) is one of the problems in integrated logistics optimizations. The LRP can be defined as follows. A feasible set of potential facility sites and locations and expected demands of each customer are given. Each customer is to be assigned to a facility, which will supply its demand. Vehicles dispatched from the facilities and operated on routes that include multiple customers carry out the shipments of customer demand. There is a fixed cost associated with opening a facility at each potential site. A distribution cost associated with any routing of vehicles includes the cost of acquiring the vehicles used in the route, and the cost of delivery operations. The cost of delivery operations is linear with the distance traveled by vehicles. The LRP is to determine the location of the facilities and the vehicle routes from the facilities to customers to minimize the sum costs of the location and distribution in this way that the vehicle capacities are not exceeded.

Many methodologies have been utilized to solve the LA problem. The most obvious methodologies are the transportation/assignment, and the linear programming formulations. Many researchers have applied integer and mixed integer formulations. Others incorporated stochastic functions to account for such distributions as demand and /or supplies are not deterministic. Baumol and Wolfe [8] solved the location problem for minimum total delivery cost with nonlinear programming. They gave a model of 0-1 mixed integer programming. The

model ensured minimum total costs. The weakness of their model is that the fixed cost and the capacity's limit are considered, which makes it possible to choose much more distribution centers. So the total cost will be increased by this method. We modify the assumptions to develop a LA model that focuses on 0-1 mixed integer programming. In this paper a heuristic algorithm based on wavelet analysis is proposed for LA, which combines the minimum wrap-net traditional method for choosing facilities. The intention of the proposed mixed method is to determine the optimal position of Distribution Center (DC) based on wavelet analysis theory. An important and instructive issue is how to eliminate noise and raise the accuracy of the solutions. First, the data is filtered by wavelet analysis, and then a heuristic algorithm is used to search for optimal (or near optimal) solutions. The method incorporates both the distance and the demands among different customers. The solution's precision is improved by wavelet analysis. As a result, customers and their chosen facilities are determined. The result of computer simulation proves that this method is effective. The rest of the paper is organized as follows. Section 2 introduces descriptions of LA and describes the mathematical model of LA. Section 3 emphasizes on the solution of LA. Section 4 shows the computational results and analysis. Finally, section 5 gives conclusions and future work.

2 Descriptions of LA

Location-Allocation (LA) is one of the methods used for problems of integrated logistics optimization. The LA can be defined as follows. A feasible set of potential facility sites and locations, and expected demands of each customer are given. The amount and location of the facilities are determined in spatial positions according to the interaction between candidate points for customers and goods distribution [9]. In fact, the main properties characterizing the facilities are their quantities, their types and the costs associated with them. A great number of objectives for LA are used to determine the optimal spatial positions and the total distance. The objectives for LA are important for planning in warehouses, plants, schools, hospitals, retail outlets and many other public structures. John Current et al. [10] give a review of LA. They put forward the standard of the objective function for classification, which consists of minimum cost, customer demand, maximum profit, and other condition. A typical description of LA problem is shown in Fig. 1.

The hypotheses for mathematical model of LA [8][9] are as follows.

- The candidate facility's position with the number $r(r=1,2,\dots,R)$ is determined, which is chosen with the maximum limit of R .

- The position and the demand represented by D_j ($j=1,2,\dots,N$) of N customers are also considered, respectively.

The factors are focused on as follows.

- the transportation cost from DC to node;
- the fixed and changeable costs from DC to node;
- The limit of DC's capacity;
- the restricted amount of DC.

Decision variables considered are as follows.

- $x_{ijk} = \begin{cases} 1 & \text{if vehicle } k \text{ goes from customer } i \text{ to customer } j \\ & i \in S, j \in S, k \in V, i \neq j, \\ 0 & \text{otherwise} \end{cases}$
- $Z_r = \begin{cases} 1 & \text{if a facility is established at site } r, r \in G \\ 0 & \text{otherwise} \end{cases}$

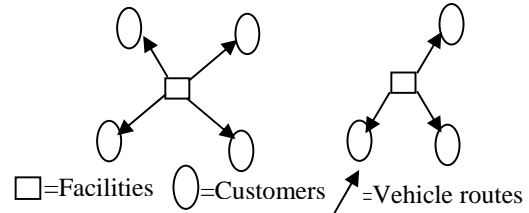


Fig 1. A typical location-allocation problem

Model parameters are given as follows.

- $G = \{r | r=1,\dots,m\}$ - the set of m feasible sites of candidate facility,
- $H = \{i | i = m+1,\dots,m+N\}$ - the set of N customers to be served,
- $S = \{G\} \cup \{H\}$ - the set of all feasible sites and customers (it is also referred to as nodes),
- $V = \{v_k | k=1,\dots,K\}$ - the set of K vehicles available for routing from the facilities,
- C_{ij} - average cost of per distance from node i to node $j, i \in S, j \in S$,
- F_r - annual fixed cost of establishing and operating a facility at site $r(r=1,\dots,m)$, which is no related to the DC's scale,
- q_j - average amount of goods demanded by customer $j, (j \in H)$,
- Q_k - capacity of vehicle $k(k=1,\dots,K)$,
- d_{ij} - distance from node i to node j ,
- V_r - changeable costs' coefficient with the candidate facilities (or called DC),
- W_r - rate of flow volume,

- θ - the exponent for flow volume, which is usually chosen 1/2,
- R - maximum quantity of the chosen facilities,
- D_j - total demand of node j .

Model of a special LA is defined as follows.

The objective function of LA is

$$f(x) = \min \sum_{i \in S} \sum_{j \in S} \sum_{k \in V} C_{ijk} X_{ijk} q_j d_{ij} + \sum_{r \in G} Z_r V_r (W_r)^\theta + \sum_{r \in G} F_r Z_r \quad (1)$$

subject to

$$\sum_{k \in V} \sum_{i \in S} X_{ijk} = 1 \quad \forall j \in H, \quad (2)$$

$$\sum_{i \in H} \sum_{j \in S} q_j X_{ijk} \leq Q_k \quad \forall k \in V, \quad (3)$$

$$\sum_{i \in S} \sum_{j \in H} X_{ijk} q_j \geq D_i \quad \forall i \in H, \quad (4)$$

$$\sum_{i \in S} X_{ipk} - \sum_{j \in S} X_{pj k} = 0 \quad \forall k \in V, p \in S, \quad (5)$$

$$\sum_{r \in G} Z_r \leq R \quad \forall r \in G, \quad (6)$$

$$X_{ijk} = 0 \text{ or } 1 \quad \forall i, j \in S, k \in V, \quad (7)$$

$$Z_r = 0 \text{ or } 1 \quad \forall r \in G. \quad (8)$$

In this model, the objective function minimizes the total costs of routing, establishing and operating the facilities. Constraints (2) ensure that each customer is visited exactly once. Constraints (3) ensure that the vehicle capacity constraints are not exceeded for any of the vehicles used in routing, while constraints (4) guarantee that customer demands are satisfied. Constraints (5) ensure that if vehicle k visits a point, it has to leave the point again (balance equation).. Constraints (6) give the limit of DC's amount. The last two of constraints are the integer constraints [11].

3 A heuristic algorithm to solve LA based on wavelet noise elimination

3.1 Data filtering based on wavelet analysis

Wavelet analysis is a signal analysis theory of a new time-frequency localization based on Fourier analysis. The wavelet analysis of signal is becoming increasingly popular within the scientific community. With respect to traditional Fourier decomposition techniques, wavelet analysis allows signals having a large bandwidth characterized by a small number of coefficients [12].

The mathematical expression of wavelet function

Definition 1: Suppose that $\Psi(w) \in L^2(R)$ ($L^2(R)$ is an actual amount of space with square-integral, i. e., signal space is of finite energy) and that its Fourier transform is $\hat{\Psi}(W)$. If $\hat{\Psi}(W)$ is satisfied with

$$C_\Psi = \int_R \frac{|\hat{\Psi}(W)|}{|w|} dw < \infty \quad (9)$$

$\Psi(w)$ is called the mother wavelet.

By zooming extension and translation, the mother wavelet $\Psi(w)$ can be obtained by wavelet sequences.

Data filtering

The signal gets decomposed into the superposition of wave packets. They are localized both in time and in frequency. These wavelet packets are obtained by rescaling a chosen Mother Wavelet $\Psi(t)$ with a proper scale to obtain a wavelet function. This allows a very efficient decomposition of spiky signals and non-stationary signals in general. In practice the popular Discrete Wavelet Transform algorithm is usually used to perform the wavelet analysis. One important application is that 1-D signal with noise used to be decomposed by wavelet analysis. A 1-D signal that contains noise can be expressed as follows:

$$s(i) = f(i) + \sigma \cdot e(i), \quad i = 1, \dots, n-1 \quad (10)$$

where $f(i)$ is the real signal, $e(i)$ is noise, and $s(i)$ is a signal with noise. In practical engineering, useful signals usually show some low-frequency signals or steady signals, and noise signals show high-frequency ones. So the process of eliminating noise is as follows. Firstly, signal is decomposed by wavelet, for instance, three-channel decomposition. Fig. 2 shows decomposition process. Because noise is usually contained in CD1, CD2, CD3 of high frequency, wavelet coefficients are dealt with in threshold. Then signal is reconstructed in order to eliminate noise. The purpose is that signal $s(i)$ eliminates restraining noise and the real signal $f(i)$ is recovered from signal $s(i)$.

Wavelet's choosing and wavelet channel's definition

The channels of wavelet decomposition greatly influence on the issue. If there is a lower channel, the high frequency in the signal leaves more, however higher channel, and filter the useful low-frequency. It is determined on the specific purpose, but experiment on the historical data to decide the channel. Meanwhile used to many experiments, wavelet 'db3' is also used to decompose one channel to eliminate noise and reconstruct the channel, from which good results can be obtained.

From Fig.3, the data of eliminated noise is smoother than that of primitive.

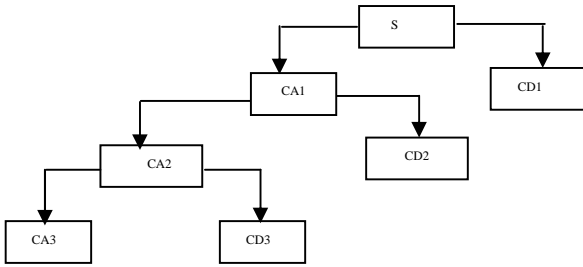


Fig.2 Three channels of wavelet decomposition

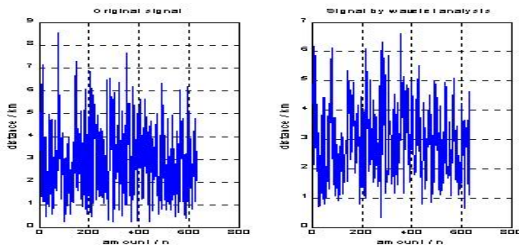


Fig.3 the comparison between original signal and filtered signal by wavelet analysis

3.2 The idea of the heuristic algorithm for LA

The fundamental idea of the heuristic algorithm for LA contains several rules. The rule for choosing a distribution center is that customers are classified into one cluster if their distances are nearest to DC. And then the customers in one cluster satisfy that the total demand of them in each DC is approximately equal to several integral multiple of single vehicle's capacity. The specific rules for choosing potential facilities are as follows.

The minimum wrap-net rule is that one distribution center is standard, which is linked to other DCs through a straight line as their perpendicular bisector. The minimum wrap is delimited on the basis of these lines of inner points in every distribution center that is served within these ranges. This rule is shown in Fig. 4.

The detailed division including served customers with different DC is given by a rule that the total demand of customers in one divided cluster is nearly equal to several integral multiple of single vehicle's capacity.

The special customers belonging to some DCs that have been categorized into one cluster may be changeable in order to arrange DC with their distributed customers more suitably.

Meanwhile, both distances between customers and demands of them are considered to have a new distributed relation. The relation indicates that special DC serves customers. The suitable facilities serving special customers are chosen by this proposed algorithm. They satisfy the minimum total distance; i. e., the distance is accumulated between customers and their facilities. Thus, the total cost is minimized.

The definition of the heuristic functions

Customers are determined to be served by DC that is nearest to them. When the goods quantity is certain, the total cost is minimized, that is to say, the total distance is the minimum one.

The heuristic function is described as follows:

$$f_1(x) = HSZ_1 = \sum_{i=1}^N MINL_i \quad (11)$$

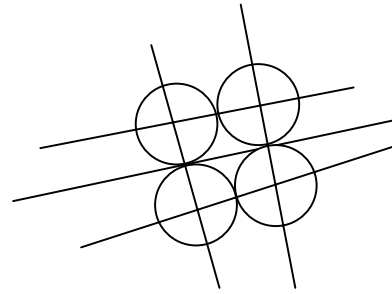


Fig.4 A simple description of minimum wrap-net

$$f_2(x) = HSZ_2 = \sum_{p=1}^i q_p = nQ_k \quad (12)$$

$$(n = 1, 2, 3, \dots, N; p = 1, 2, \dots, i)$$

$$L = \sum_{i=1}^N d_i \quad (13)$$

where $MINL_i$ is the minimum one of the distances from customer i to m DCs, N is the amount of customer points, d is the distance from any customer i to any DC.

3.3 The analysis for LA based on a heuristic algorithm by wavelet analysis

The customer aggregate C is divided into clusters by the euclidean measure based on the minimum wrap-net method. Optimizing facility location is performed in the following way, which has been extensively analyzed and used in location theory to approximate distances between two spatial coordinates. For the sake of simplicity, it is assumed that each facility has its own coordinates set of m candidate sites for facility location, and no two facilities share the same location, which are denoted by $PF_1(X_1, Y_1)$, $PF_2(X_2, Y_2), \dots, PF_i(X_i, Y_i), \dots, PF_m(X_m, Y_m)$. Every two

coordinates of candidate sites for facility locations are denoted by $PF_i(X_i, Y_i)$ and $PF_j(X_j, Y_j)$, and which satisfy the following expression:

$$R_{ij} = 1/2[(X_i - X_j)^2 + (Y_i - Y_j)^2]^{1/2} \quad (14)$$

where R_{ij} is half of the distance between $PF_i(X_i, Y_i)$ and $PF_j(X_j, Y_j)$.

Rules for classification are given as follows.

Rule 1 $r_{ip} < R_{ij}$ (15)
 C_p can only be allocated to PF_i if formula (15) is true.

Rule 2 Customers $C_1, C_2, C_3, \dots, C_b, C_m$ are classified in O_i , if $C_1, C_2, C_3, \dots, C_b, C_m$ are served by DC i . The remaining customers C_m are moved into the DC that is nearest to DC i . This minimum wrap-net method for clustering analysis incorporates the demand into relative distances among customers. It is useful for solving practical LA, in which the distance related to demand with different customers is considered at the same time.

Rule 3 The coordinates of customers round a facility satisfy that the distance between the nearest two customers is less than that between the sub-nearest two points (Sub-nearest two points means that the two points are not included in a cluster by measure of distance).

Rule 4 If $r_{jq} > \max R_{ij}$ (16)
 where r_{jq} is the distance from any customer C_q to any facility PF_j , $\max R_{ij}$ is the maximum one of half distances between any two facilities. Under this rule, either these customers will not be served, or else the candidate facility location will be abolished. Last two rules only relate to abnormal customer points.

In the above rules, N is the number of customers, $C = [C_1, C_2, \dots, C_N]$, $C_1(x_1, y_1), C_2(x_2, y_2), \dots, C_p(x_p, y_p), \dots, C_N(x_N, y_N)$ express customer points and their coordinates, respectively. r_{ip} is the distance between any candidate facility PF_i and any customer p . Tangent circles $O_1, O_2, \dots, O_b, O_p, \dots, O_m$ are drawn with the radius R_{ij} , where $R_1, R_2, \dots, R_i, R_j, \dots, R_m$ are their radiuses, respectively.

3.4 Steps of the heuristic algorithm for LA based on wavelet analysis

Step 1 The data is filtered by wavelet analysis. Wavelet 'db3' is used to decompose one channel to eliminate noise and reconstruct it best.

Step 2 Let $m=R$ and points of customers are chosen from potential R facilities. Initial calculations for R_{ij} and r_{ip} and heuristic function's value are performed. And heuristic function's value is given to $bestHSZ$, which referred to as the best heuristic function's value.

Step 3 If $r_{ip} < R_{ij}$, the customer p falls into cluster O_i . Remember the minimum function value: $opt_f = bestHSZ$. Otherwise, return to step 1.

Step 4 Calculate the total demand satisfying that it is the integral multiple (several times) of single vehicle's capacity in any cluster as precisely as possible.

Step 5 If $f < bestHSZ$, and $opt_f = f$. Let $m=R-1$, that is to say, one potential facility is rejected. And repeat step 1 to step 3, calculate R_{ij} and r_{ip} and heuristic function's value.

Step 6 If $f^* < bestHSZ$, and $opt_f = f^*$. Goto step 4, and decide whether the cycle is stopped or not.

Note: The rule for reducing potential facilities is defined. The cost increased in transportation for customer is less than the cost of establishing and operating one facility. If the rule holds true, one facility is cut off.

4 Analysis of the computational results

In the following example six potential facilities and thirty customs with different demands are analyzed. The same type of distribution vehicle that has a carrying capacity of 30 tons serves each customer. The annual cost of establishing and operating a facility F_r is 160 Yuan. The transportation cost C_{ij} is 2 Yuan per ton, per km. The data shown in Fig.3 is used to search the optimal (or near optimal) solutions by this algorithm. The programming is done by VC++. The results of clustering analysis based on wavelet analysis with minimum wrap-net method and running 3000 times are given in Fig. 5 and table 1.

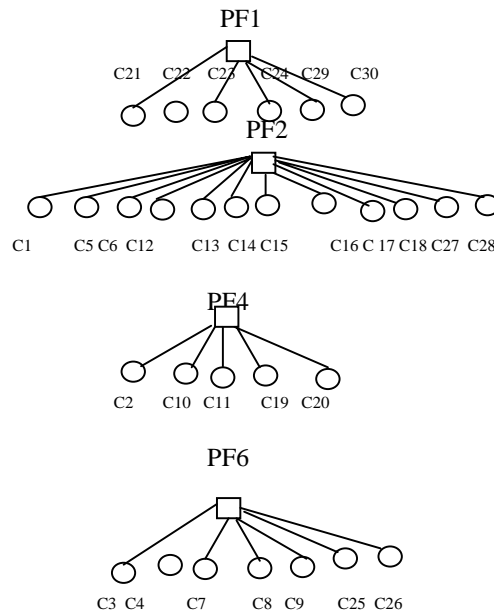


Fig.5 Results by a heuristic algorithm based on wavelet analysis with minimum wrap-net

| Problem scale | Results by heuristic algorithm based on with minimum wrap-net | Function value |
|---------------|--|----------------|
| 6/30 | ①PF1- C21,C22, C23,C24,C29,C30, ②PF2- C1,C5, C6, C12, C13, C14, C15, C16, C17, C18,C27,C28, ③PF4- C2,C10,C11,C19,C20, ④PF6- C3,C4,C7,C8,C25, C26. | 7401.49 |

Table1 Simulation results based on wavelet noise elimination

From the simulation results, the number of potential facilities that are chosen depends on special conditions. When the location of customers are widely scattered and the demand of customers is very large, the four potential facilities are all chosen, and each chosen facility serves its special customers who are nearest to it. The simulation results are shown in Fig. 5 and table 1. The value of objective function $f(x)$ is 7401.49 Yuan.

5 Conclusions

In this paper, the heuristic algorithm based on wavelet analysis for mixture choosing facilities is proposed. It combines customer's demand into minimum wrap. This method can solve the LA problem with many potential facilities and different demand customers. A classification is given out, in which customers are related to their divided DC. Using wavelet analysis technique, the solution's precision is improved. This method has contributed to the practical application of LA in integrated logistics. The proposed method for LA provides a way to solve LRP. The application to other combinatorial optimization problems should be investigated in integrated logistics.

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