# Shape Part Correspondence Using Chance Probability Functions

**Boaz J. Super** [1]
**Motorola Labs**
**Schaumburg, IL USA**

## Abstract

*This paper presents a direct method for finding corresponding pairs of parts between two shapes. Statistical knowledge about a large number of parts from many different objects is used to find a part correspondence between two previously unseen input shapes. No class membership information is required. The knowledge-based approach is shown to produce significantly better results than a classical metric distance approach. The potential role of part correspondence as a complement to geometric and structural comparisons is discussed.*

**Keywords:** part correspondence, shape matching, chance probability functions, knowledge-based matching

## 1. Introduction

Recent research on shape matching has emphasized two shape matching problems. In *geometric shape comparison*, correspondences are found between points or other low-level spatial data of the shapes [1,2,3,7]. In *structural shape comparison*, shape parts and their relationships are matched [6,10].

In this paper we explore two ideas. First, we propose a third shape matching problem, *part correspondence*, that is complementary to both geometric and structural matching. A part correspondence is a mapping between two sets of parts. Compared with points and other low-level geometric data typically used for geometric matching, complex parts are less ambiguous and more salient. They also lend themselves naturally to handling occluded and articulated shapes, and to indexing object databases by parts [12]. In contrast with structural matching [5,6,10], part correspondence matches only parts and not relationships. Part correspondence can be viewed as a type of shape comparison intermediate between geometric and structural comparisons.

The second idea explored here is that the widely-studied problem of matching shapes in isolation – that is, without considering knowledge about all shapes – is an artificial problem. This view is motivated by current understanding that biological vision systems exploit knowledge in performing visual processing tasks. A specific contribution of this paper is the use of knowledge of a large set of shape parts, from many different objects, to define a probabilistic measure of part difference. The central empirical demonstration of this paper is that when a geometric measure of part difference is used, a simple greedy algorithm for part correspondence fails; but when the corresponding probabilistic measure of part difference is used, the same algorithm succeeds. The large difference in outcome is due entirely to the use of the probabilistic part difference measure instead of the geometric one; everything else is held constant.

No knowledge of shape classes is required by the method. That is, although shape examples are required, it is not necessary that the examples be labeled or classified.

To prevent confusion, note that the

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[1] Work performed while the author was at the University of Illinois at Chicago.

statistical model presented in this paper is not related to the class of methods known as "statistical shape modeling" (SSM) [4]. The present method was developed as a bootstrapping tool for a project on automatically learning SSMs (shape class models) from examples. It was thus designed for the problem of matching shapes represented as contours, and not for the problems of matching shapes to images [4,5] or images to images. The method works on sets of contour segments, which can come from shapes consisting of multiple open or closed contours. However, only single-contour shapes are supported in our current implementation.

The statistical model used here is embodied in a set of *chance probability functions* (CPFs) [13]. The term "chance" in this context refers to a chance (null) hypothesis. In previous research, we introduced CPFs based on whole planar shapes in order to improve accuracy of shape retrieval and classification [13,14]. However, the previous work was applicable only to matching whole, unoccluded shapes, and gave no information about the internal structure of the shapes. In this paper we use part-based CPFs instead of shape-based CPFs, and extend the knowledge-based matching idea to the new problem of finding part correspondences between two shapes.

# 2. Part Correspondence Method

This section describes the method. Section 2.1 summarizes the statistical model; Section 2.2 describes the generation and representation of parts; and Section 2.3 describes the use of the model to find part correspondences.

## 2.1 Chance probability functions

This section briefly summarizes the statistical model that was presented in detail in [13,14]. The basic intuition is that the distribution of shape parts in a high-dimensional part representation space is not uniform. Some shape parts are distinctive, appearing in one or a few object classes. This is more likely when the parts are complex. Other parts are relatively generic, appearing in many object classes. This is more likely when the parts are small, simple, or both. Because of the nonuniformity of the part distribution, a given metric distance between a pair of parts has different significance in high- and low-density regions of part space. In a high density region two parts may be more likely to be near each other by accident, rather than because they are corresponding parts of similar objects; the opposite is true in a low density region.

Instead of measuring the similarity of two parts by a distance value $D$ (e.g., Euclidean distance) in the representation space, we will measure their similarity by the probability of obtaining the observed value of $D$ by chance. Specifically, given a particular part $p$ and a second part $q$, we measure the probability of obtaining a part within distance $D(p,q)$ of $p$ by chance. The chance hypothesis in this case is defined as random selection, with equal probability, of a part from the set of all parts of all example shapes of all classes in a given database of shape examples. The less likely that a value of $D$ less than or equal to $D(p,q)$ would occur by chance, the more similar the two parts are considered to be. The term "chance probability" will be used as shorthand for "the probability of an event occurring as the result of the chance hypothesis."

The advantages of the statistical method in comparison to other statistical methods used in computer vision may be summarized as follows: unlike class-based Bayesian inference, no class information is needed; unlike nonaccidentalness, no feature combination models are needed; and unlike kernel-based density estimation, no free parameters are needed in the training stage. A detailed discussion of these comparisons is given in [14].

The basic element of the proposed method is the *chance probability function* (CPF). There is one such function for each part, $p$, of a shape to be matched. The value of this function at metric distance value $D_0$ in the representation space is the probability of getting a part within $D_0$ of $p$ by chance. It is estimated by

$$CP_p(D_0) = |\{ D(p,p') \leq D_0 : p' \in S \}| / |S| , \quad (1)$$

where $S$ is a large set of parts from example shapes of multiple classes, $|\,.\,|$ is the set cardinality function, $D$ is a distance measure in a part representation space, and $p'$ is an arbitrary part in $S$. The CPF is a cumulative probability distribution; the term chance probability function is used to indicate that it is specifically the distribution with respect to the chance hypothesis. The CPFs are computed for each part of the shape to be matched, using a large part database as the set $S$. Then, for each part $p$ in one shape to be matched and each part $q$ in the other shape to be matched, the CPF of $p$ is used to map the geometric difference $D(p,q)$ to the probability of that difference, $CP_p[D(p,q)]$. The details are available in [13,14]; in particular, the CPFs are represented as look-up tables with a high degree of compression (e.g., 100-to-1 compression), they are linearly interpolated to change them from discrete to continuous, and the chance probabilities are symmetrized:

$$D_{CP}(p,q) = \max\{CP_p[D(p,q)], CP_q[D(p,q)]\}. \quad (2)$$

The central experiment presented in this paper is a direct comparison of the metric distance $D(p,q)$ with the probabilistic part difference measure $D_{CP}(p,q)$. All other factors – the part generation method, the part representation method, and the matching algorithm – are held constant.

## 2.2 Part generation and representation

The part correspondence method is independent of the part generation and representation methods used. This paper uses simple, rapidly computed methods. More sophisticated methods may be substituted, but we have not found that necessary.

Shapes are represented by closed or open planar contours. We discuss the closed contour case; the open contour case is nearly identical. Let $c$ denote a contour. A set of key points $K = \{K_1,...,K_n\}$ is found. In the current implementation, key points are significant positive local maxima and negative local minima of curvature [14] (Fig. 1a). The set of parts $S_I$ is defined to be the set of all intervals between distinct key points on $c$. Let $[K_i,K_j]$ denote the restriction of $c$ to the oriented interval between key points $K_i$ and $K_j$. (Note that on a closed contour, $[K_i,K_j] \cup [K_j,K_i] = c$.) There are $n(n-1)$ parts. Most shapes we have examined (about 1,500) have between 10 to 40 key points, so in practical terms $|S_I|$ is effectively bounded.

Each part in $S_I$ is represented in an invariant reference frame by translating, rotating, and scaling the part so that $K_i \rightarrow (0,0)$ and $K_j \rightarrow (1,0)$ (Fig. 1b). Within the invariant reference frame, every part $p$ is resampled at uniform arc length intervals by $\alpha$ points $\mathbf{x}_1,...,\mathbf{x}_\alpha$, where $\mathbf{x}_i = (x_i,y_i)$. A sum of squared distances (SSD) difference measure between parts $p$, $p'$ is defined by

$$D_{SSD}(p,p') = \sum_{i=1}^{\alpha}(\mathbf{x}_i - \mathbf{x}'_i)(\mathbf{x}_i - \mathbf{x}'_i)^T . \quad (3)$$

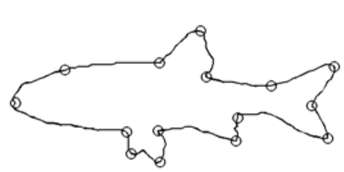
Each part can also be viewed as a concatenated vector $\mathbf{v} = (\mathbf{x}_1,...,\mathbf{x}_\alpha)$ in a $2\alpha$-dimensional part space; then $D_{SSD}(p,p')$ is squared Euclidean distance in that part space. It is important to note that the match is component-by-component vector matching; there is no attempt to solve the point correspondence problem in its usual sense of determining which points in $p'$ are closest to, or correspond best to, points in $p$ [2,3].

## 2.3 Part correspondence

This section presents the definition of a part correspondence between two shapes and a method for finding that correspondence.

Let $S_E$ denote the set of *elementary parts* of a shape, defined by adjacent key points, and let $S_C$ denote the closure of $S_E$ under set union. Then $S_E \subseteq S_I \subseteq S_C$.

Elementary parts are not generally useful for part correspondence because they are often too simple to match unambiguously; furthermore, corresponding elementary parts might not exist

(a)


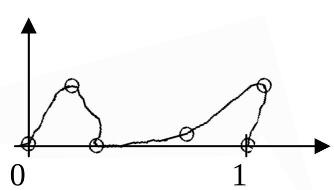
(b)



Figure 1. (a) Key points are positive local maxima and negative local minima of curvature. (b) An example of a normalized part (from front of dorsal fin to center notch of tail).

Figure 2. Two examples from each of the ten object classes used to generate the parts database.

due to contour segmentation differences. Instead, given an elementary part π, we find the part $p$ in the set of all parts containing π, such that $p$ has the best match to a part $p'$ in the other shape. The part $p$ can be thought of as the adaptively selected context for π that can be most confidently matched to a part in the other shape, in the sense of having the least probability of occurring by chance. The part correspondence will be constructed from a set of adaptively selected context parts that cover the set of elementary parts. Since multiple elementary parts can share the same containing parts, redundant part comparisons can be avoided by using the following greedy algorithm: (1) select the best matching pair of parts; (2) delete subparts of that pair; (3) output the portions of the pair that cover sections of the contour that have not yet been matched; (4) repeat until no more matches can be performed.

## 3. Experiments

The central thesis of this work is that reliable part correspondences between two shapes can be found directly when a knowledge base is available. The knowledge used is statistical information about the distribution of parts in part space as captured in the chance probability functions. To demonstrate the approach, we will compare it to a classical approach using Euclidean distance in part space.

The database of parts used in the experiments was generated by choosing 10 classes from a large database of silhouette shapes [8]. Five shapes were randomly selected from each class. (Fig. 2 shows two examples of each class.) The mirror reflections of the shapes were also used, to yield a total of 100 shapes. The part generation procedure described in Section 2.2 was applied to each shape, yielding a database of 19,220 parts. At runtime, the part database was used to compute the chance probability function $CP_p$ for each part $p$ in the two input shapes to be matched. In all of the experiments, the input shapes were different from those used to generate the part database.

The part correspondences were generated using the algorithm described in Section 2.3 with two different shape difference measures: (1) squared Euclidean distance between the part vectors in 2α-dimensional space ($\delta = D_{SSD}$), and (2) chance probabilities ($\delta = D_{CP}$). To handle reflections, $s'$ was matched to both $s$ and the reflection of $s$; the version with the lower total match score was used.

Fig. 3a shows two new elephant shapes (i.e., not among those used to generate the part database). Fig. 3b shows the key points that define the endpoints of extracted parts. When $\delta = D_{SSD}$, the part correspondence consists of 20 part pairs. The best four matches are shown in Fig. 3c. All four of these matches are incorrect. When $\delta = D_{CP}$, the complete part correspondence consists of four part pairs, shown in Fig. 3d. Each of the part correspondences is correct. With squared Euclidean distance, the matched parts are

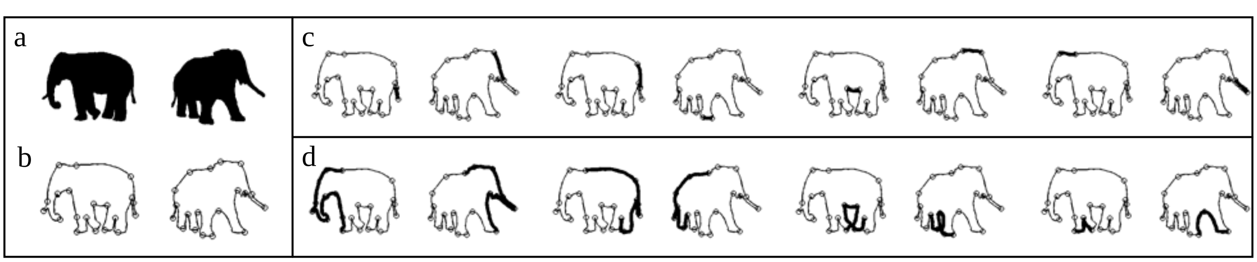



Figure 3. (a) Two previously unseen elephant shapes. (b) Detected key points. (c) The best four part matches using $D_{SSD}$. Each is incorrect. (d) The complete part correspondence found using $D_{CP}$. All correspondences are correct.

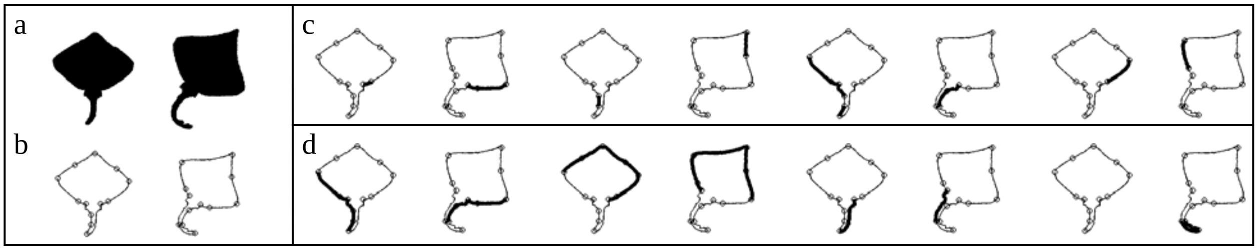



Figure 4. (a) Two previously unseen ray shapes. (b) Detected key points. (c) The best four part matches using $D_{SSD}$. The first three are incorrect. (d) The complete part correspondence found using $D_{CP}$. All correspondences are correct. Notice that in (d) the extra structure in the right ray's tail is correctly found not to match any part of the left ray.

smaller, less distinctive, and often incorrectly corresponded. With chance probabilities, the parts are larger, more complex, and correctly corresponded, even though some of the corresponding parts are quite different geometrically.

It must be emphasized that the method does not use class information in any way. All of the information resides in the CPFs which are computed from a single set of parts from all classes. The parts in the database are not labeled by class nor by the object they came from.

Fig. 4a shows two previously unseen ray shapes. Fig. 4b shows the key points. When $\delta = D_{SSD}$, the part correspondence consists of 10 part pairs. The best four matches are shown in Fig. 4c. Three of these are incorrect. (The shapes are matched under reflection because the rays' tails curve in opposite directions, so corresponding parts should occur on opposite sides.) When $\delta = D_{CP}$, the complete correspondence consists of four part pairs (Fig. 4d). Each of them is correct, including the null match for the structure on the right ray's tail that does not have a counterpart on the left ray's tail. This example also demonstrates the rotation invariance of the method.

Fig. 5 shows similar results for two previously unseen tree shapes. This example shows correct matching of parts even in the presence of repetitive structure. The method implicitly adapts the context to obtain unambiguous matches since these are less likely to occur by chance.

Novel shape pairs of the remaining seven object classes used to generate the parts database produced similar results. Although the chance probability method occasionally produced incorrect part correspondences, and SSD occasionally produced correct correspondences, the chance probability model always did as well as or better (usually much better) than SSD.

The time required to compute a complete part correspondence between two shapes varied according to the complexity of the shapes. Times ranged from 6 to 51 s with an average of 22 s on a 1.8 GHz Pentium PC running interpreted Matlab. Much of the computational time is spent computing the CPFs, currently done at runtime. Speeding up the method by precomputing information about the parts database is one area for future research.

We also tested the possibility that a modified SSD measure normalized by the arc lengths of the parts would perform better than $D_{SSD}$. A second experiment (not shown) demonstrated that normalized SSD also produced many incorrect part correspondences.

(a) (b) (c) (d) (e)

Figure 5. Part correspondence in the presence of repetitive structure. (a) Two previously unseen tree shapes. (b) Key points. (c-e) The complete part correspondence found using $D_{CP}$.

## 4. Discussion

Many knowledge-based approaches use either models of classes or examples labeled by class (e.g., [4,11]). The present approach is different in that no information about classes is used: the knowledge consists of the density of parts in part space across multiple classes. That this relatively low-level knowledge can be so effective in disambiguating matches is the most important demonstration of this paper.

We compared a geometric difference measure $D_{SSD}$ and a probabilistic difference measure $D_{CP}$ using the same algorithm. The results, together with earlier results on shape retrieval and classification using CPFs [13,14], suggest that context in the representation space (provided by $D_{CP}$) may be more important than pure geometric matching ($D_{SSD}$) in determining shape similarity.

For a given elementary part, the method adaptively chooses the size of the context in the shape that maximizes the confidence of the match in the sense of minimizing the probability of its occurring by chance. This represents an advance over matching methods that disambiguate matches using global context [1] or local contexts defined by a fixed number of feature points [7]. The adaptive behavior emerges automatically when chance probabilities are substituted for geometric differences in the greedy algorithm. The chance probabilities provide the connecting mechanism whereby the context of a part in the representation space determines the context of that part within its shape.

It is important to note that the present method does not solve a point correspondence problem between parts. The matching of parts to find $D_{SSD}$ (which is then used to find $D_{CP}$ using the CPF lookup tables) is fixed, component-by-component vector matching: there is no attempt to solve the correspondence problem in the usual sense of determining which point of one part best matches which point of another part. (I.e., the 'correspondence' is independent of the data.)

It is also important to note that the method does not explicitly model or match part relationships between the two shapes. An intriguing area of future research is to explore the use of part correspondences to reduce the size of the search space in structural matching approaches.

## 5. Conclusion

This paper advocated two ideas: (1) part correspondence as a new type of shape comparison, and (2) knowledge-based shape matching. The paper presented a class-free model in which the knowledge used to match the parts of two shapes is embodied in chance probability functions computed from a large set of parts of many different shapes. The paper also presented a method for finding a set of part correspondences between two shapes. A side-by-side comparison of Euclidean distance with the probabilistic difference measure showed that the power of the method is due to the chance probability model. Part correspondence is a potentially valuable form of shape matching, complementary to both geometric and structural matching.

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## References

[1] S. Belongie, J. Malik, and J. Puzicha, Shape Matching and Object Recognition Using Shape Contexts. *IEEE Trans. Pattern Analysis and Machine Intelligence* **24**(4), 509-522, 2002.

[2] P. Besl and N. D. McKay, A Method for Registration of 3-D Shapes. *IEEE Trans. Pattern Analysis and Machine Intelligence* **14**(2), 239-256, 1992.

[3] H. Chui and A. Rangarajan, A New Point Matching Algorithm for Non-Rigid Registration, *Computer Vision and Image Understanding* **89,** 114-141, 2003.

[4] T. F. Cootes, C. J. Taylor, D. H. Cooper, and J. Graham, Active Shape Models - Their Training and Application, *Computer Vision and Image Understanding* **61**(1) 38-59, 1995.

[5] P. F. Felzenszwalb and D. P. Huttenlocher, *Efficient Matching of Pictorial Structures*, *IEEE Conf. Computer Vision and Pattern Recognition*, II:66-73, 2000.

[6] Y. Gdalyahu and D. Weinshall, Flexible Syntactic Matching of Curves and Its Application to Automatic Hierarchical Classification of Silhouettes, *IEEE Trans. Pattern Analysis and Machine Intelligence* **21**(12), 1312-1328, 1999.

[7] C. Grigorescu and N. Petkov, Distance Sets for Shape Filters and Shape Recognition, *IEEE Trans. Image Processing* **12**(10) 1274-1286, 2003.

[8] L. Latecki, R. Lakämper, and U. Eckhardt, Shape Descriptors for Non-rigid Shapes with a Single Closed Contour. *Proc. IEEE Conf. Computer Vision and Pattern Recognition*, 2000.

[9] F. Mokhtarian, S. Abbasi, and J. Kittler, Efficient and Robust Retrieval by Shape Content through Curvature Scale Space, in A. Smeulders and R. Jain (Eds.), *Image Databases and Multi-Media Search*, World Scientific, New Jersey, pp. 51-58, 1997.

[10] T. B. Sebastian, P.N. Klein, and B. B. Kimia, Recognition of Shapes by Editing Their Shock Graphs, *IEEE Trans. Pattern Analysis and Machine Intelligence* **26**(5) 550-571, 2004.

[11] K. B. Sun and B. J. Super, Classification of Contour Shapes Using Class Segment Sets, *IEEE Conf. Computer Vision and Pattern Recognition (CVPR)*, pp. 727-733, 2005.

[12] B. J. Super, Fast Retrieval of Isolated Visual Shapes, *Computer Vision and Image Understanding* **85**(1) 1-21, 2002.

[13] B. J. Super, Learning Chance Probability Functions for Shape Retrieval or Classification, *Proc. IEEE Workshop on Learning in Computer Vision and Pattern Recognition*, Washington DC., June 2004.

[14] B. J. Super, An Effective and Fast One-to-Many Shape Matcher, *International Journal of Pattern Recognition and Artificial Intelligence*, in press.