A New Algorithm of Phase Unwrapping

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Abstract:Phase unwrapping is one of important task in synthesizes aperture radar (SAR) interferometric measurement data processing procedure. Accuracy of it will directly affects precision of the interferometric measurement result. Through the experimental study, this paper analyzes principle and the characteristic of two kind of main current phases unwrapping algorithm, then conbines respective merit and introduce one kind of effective even better two-dimensional phase unwrapping algorithm.

Key word: InSAR, Phase Unwrapping, Least-Squares, Region-Growing

1. Introduction

The earliest applications of synthetic aperture radar (SAR) interferometric measurement techniques mostly focused on measurement of surface topography from interferograms. It is a powerful technique allowing the generation of digital elevation models (DEM). In order to get a high quality DEM, we must have a low-noise interferometric image. To unwrap the phase, it is necessary to combine two single complex views which obtained from different observation angles in the same scene. In a low-noise case the unwrapping would be easy, however, local errors which is due to noise will result in global errors of unwrapping.

Phase unwrapping is a computational process that a surface φ (often representing "phase") is reconstructed from its so-called "wrapped" form ϕ, $-\pi < \phi \le \pi$. In the absence of noise, $\phi(x)$ is equal to $\varphi(x) + 2\pi k(x)$, where $K(x)$ is an integer-valued function. It is an essential and important part of InSAR measurement techniques. The unwrapping process aimed at providing an estimation of the actual phase function φ through the given wrapped function ϕ. It turns out to be a difficult problem.

Since Goldsterin in 1988 proposed a phase unwrapping algorithm so-called Branch-Cut, various algorithms based on the InSAR phase unwrapping are unceasingly introduced. These methods approximately may divide into two groups: Local Phase Unwrapping (Path Integral) and Global Phase Unwrapping (Least-Squares) method.

Path Integral method is implemented by firstly identifying the locations of all residues in an interferogram and then connecting the residues with branch cuts to prevent the existence of integration paths that can encircle unbalanced numbers of positive and negative residues. Because of the terminology of branch cuts and the dendritic appearance of the complete set of cuts, the interconnection of the residues was referred to as growing "trees"[3]. Only integral numbers of cycles are added to the measurements to produce the result. The resulting unwrapping in the cycles is very accurate. This type of method is powerful in the local reconstruction of the unwrapped phase namely local exactness.

In Least-Squares method (Fried, 1977; Hudgin, 1977; Hunt, 1979), unwrapping is achieved by minimizing the mean square deviation between the estimated and the unknown neighboring pixel differences of the unwrapped phase. Least-squares method is very efficient computationally when they make use of fast Fourier transform techniques (Takajo and Takahashi, 1988; Ghiglia and Romero, 1994).

2. Two Kinds of Phase Unwrapping Methods

2.1 Path Integral Method

Main thought of path integral method is to choose integral path by using the different method [1]. In path integral method, the accuracy of the result depends on choice of the path to perform the unwrapping. The region-growing algorithm minimizes unwrapping errors by starting at pixels of high data quality and proceeding along dynamic paths where unwrapping confidence is high. Areas that are difficult to unwrap are then approached from different directions[2]. Thus, the algorithm is able to correct unwrapping errors to a certain extent and prevent errors from propagating.

The algorithm achieves these goals in the following ways(Fig. 1).

(1) Unwrapping is carried out concurrently in a number of regions. A region is started from a seed where the phase is locally smooth and allowed to grow outwards along controlled data-dependent paths during the unwrapping procedure;

(2) Each pixel is unwrapped based on slope predictions made from its unwrapped neighbors. The predictions allow phase changes between two adjacent pixels larger than π;

(3) Information from as many directions as possible is used to unwrap each pixel. This mitigates the effect of errors in the individual prediction directions;

(4) The reliability check based on the consistency of phase predictions is applied to each unwrapping attempt to validate (or temporally disallow) the proposed unwrapping value;

(5) Tolerance of the reliability is gradually relaxed to allow as many pixels as possible to be unwrapped while keeping the unwrapping consistency above a specified level;

(6) When regions grow together, an attempt is made to join them by trying many different connecting paths. Consistency checks are applied to ambiguity numbers of the pixels that join the regions. The ambiguity numbers of the merged regions are adjusted accordingly[3].

To provide reliable phase unwrapping, a phase prediction can be made from each of the N_μ unwrapped neighbors, N_μ is generally required to be larger than one, except near a seed. The predictions are made along lines, as illustrated in Fig. 1, where $N_\mu = 4$. The prediction ϕ_k^p is formed from the kth unwrapped neighbor, and it is either a linear or constant prediction, depending on the number of unwrapped pixels along the prediction line, as follows.

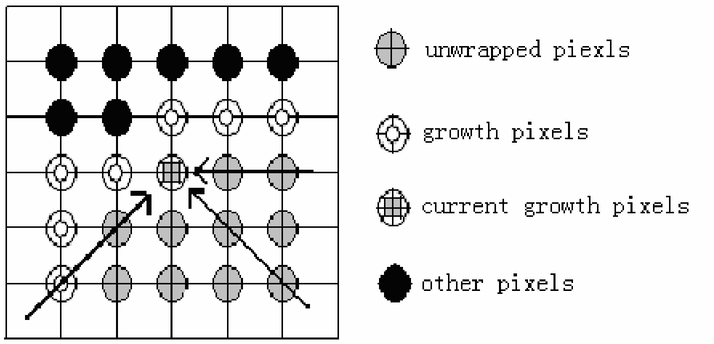

Fig. 1 Growth pixel and its neighbors showing phase prediction directions

(1) If two unwrapped pixels are available along the prediction line, a linear prediction is used

$$\phi_k^p = 2\phi[k] - \phi[k'] \qquad (1)$$

where k' stands for the next unwrapped pixel along the prediction line.

(2) Otherwise, the prediction is simply given by the phase value at pixel k

$$\phi_k^p = \phi[k] \qquad (2)$$

Then, a composite prediction ϕ^p is formed as a weighted average of the N_μ individual predictions

$$\phi^p = (\sum_{k=1}^{N_\mu} \omega_k \phi_k^p) / (\sum_{k=1}^{N_\mu} \omega_k) \qquad (3)$$

where ω_κ is one for case 1 because it is more reliable and 0.5 for case 2. The weights are shown as the numbers beside the corresponding prediction line in Fig.1.

The phase prediction (3) is used to attempt to unwrap the growth pixel. The proposed unwrapped phase value ϕ_μ at the growth pixel is computed as

$$\phi_\mu = \varphi + 2m\pi \qquad (4)$$

where φ is the wrapped phase at the growth pixel and ,the ambiguity number, is

$$m = \text{int}(\frac{\phi^p - \varphi}{2\pi}) \qquad (5)$$

where $\text{int}(x)$ is the integer closest to x. Whether the result of the unwrapping attempt is accepted or not depends on a measure of reliability discussed below.

The merit of region-growing algorithm lies in that unwrapping is carried out concurrently in a number of regions. Its speed is fastly. It considers simultaneously the many directions unwrapped phase information, and reduces the effect of direction error which forecasts individually. Its demerit is that phase in the high-noise regions can't be unwrapped and forms "isolated island". It will cause the unusable entire picture.

2.2 Least-Squares Method

Least-Squares method completely differs from Path Integral method. Its main thought is to minimize the distance between the discrete gradient of unwrapped phase and the discrete gradient estimated from the value of the wrapped phase. Basic principle as follows[4]:

We denote the wrapped values by $\psi_{i,j}$, and the sought-after unwrapped phase values by $\varphi_{i,j}$, with

$$\begin{aligned} &\psi_{i,j} = \varphi_{i,j} + 2k\pi, && k \in Z \\ &-\pi < \psi_{i,j} \le \pi && \text{i=0}\cdots\text{M-1, j=0}\cdots\text{N-1} \end{aligned} \tag{6}$$

The differences between the partial derivatives of the wrapped phase are defined as

$$\begin{aligned} &\nabla^x_{i,j} = W\{\psi_{i+1,j} - \psi_{i,j}\} \quad \text{i=0}\cdots\text{M-2,j=0}\cdots N-1 \\ &\nabla^x_{-1,j} = -\nabla^x_{0,j} \qquad \nabla^x_{\text{M-1,j}} = -\nabla^x_{M-2,j} \\ &\nabla^y_{i,j} = W\{\psi_{i,j+1} - \psi_{i,j}\} \quad \text{i=0}\cdots\text{M-1,j=0}\cdots\text{N-2} \\ &\nabla^y_{i,-1} = -\nabla^y_{i,0} \qquad \nabla^y_{\text{i,N-1}} = -\nabla^y_{i,N-2} \end{aligned} \tag{7}$$

where W is the wrapping operator that wraps the phase into the interval (-π, π). The differences between the partial derivatives of the solution $\phi_{i,j}$ and those in (7) can be minimized in the least squares sense, by differentiating the sum

$$\sum_{i=0}^{M-2}\sum_{j=0}^{N-1}(\phi_{i+1,j} - \phi_{i,j} - \nabla^x_{i,j})^2 + \sum_{i=0}^{M-1}\sum_{j=0}^{N-2}(\phi_{i,j+1} - \phi_{i,j} - \nabla^y_{i,j})^2 \tag{8}$$

The least squares phase unwrapping problem is to find the solution $\phi_{i,j}$ that minimizes the sum of (8). The least squares solution to this problem yields the following equation:

$$(\phi_{i+1,j} - 2\phi_{i,j} + \phi_{i-1,j}) + (\phi_{i,j+1} - 2\phi_{i,j} + \phi_{i,j-1}) = \rho_{i,j} \tag{9}$$

where $\rho_{i,j}$ is the 'phase Laplacian' defined by

$$\rho_{i,j} = (\nabla^x_{i,j} - \nabla^x_{i-1,j}) + (\nabla^y_{i,j} - \nabla^y_{i,j-1}) \tag{10}$$

Equation (9) is the discrete version of the Poisson's partial differential equation (PDE), $\nabla^2\phi = \rho$.

Direct methods based on the fast Fourier transform (FFT) or the discrete cosine transform (DCT) can be applied to solve the unweighted phase unwrapping problem[5]. However, in the weighted case, iterative methods should be adopted. The classical iterative method for solving the linear system is the Gauss-Seidel relaxation, which solves (9) by simple iteration until it converges. The Weighted Multi-Grid method is a very efficient and fast one .

Through the principle from Least-Squares method, we may know it is one kind of global phase unwrapping method. This method only can guarantee that the phase unwrapping result is quite good as a whole. If the phase is affected by noise, the phase slopes are always underestimated, and the method cannot reconstruct the full height of the underlying topography. These errors from decorrelated parts of the image also propagate into areas with high coherent. The result is a global distortion of the final absolute phase. This problem cannot be solved by this kind of method.

3. Combination of Two Methods

Analyzing the principle and the thought of two kinds of method, we can find that they are entirely different and have complementarity.

The propagation of error using Least-Squares algorithm can now be reduced by Region-growth algorithm. Because a local method always gives a correct solution, we are able to estimate and to correct exactly the error of the global algorithm by the means. After reaching the final absolute phase, we can simply be abstained by adding the unwrapped error matrix to Least-Squares solution.

The parts of account of high-noise which cannot be unwrapped by Region-Growth algorithm can be unwrapped by Least-Squares algorithm. Because unwrapped errors using Least-Squares algorithm in these regions are smaller than ones using Region-Growth algorithm. In high-noise areas Least-Squares method is more powerful. By unwrapping iteratively between the original phase and the preliminary result, it is possible to reach locally the correct phase gradient even in extremely low coherent areas. To bate the problem of the underestimate and allow a faster convergence to the error slope, it seems necessary to reduce the phase noise by filtering. We can observe that this method usually has no further improvement of the solution after 16 iterations.

Another important step is to minimize the coherence threshold of the Region-Growth method. The used coherent map has strong influence on the unwrapping path chosen by Region-Growth method. As we found out the best results can be obtained with coherent map calculated directly from phase values using(12).

$$\gamma = \langle E\{e^{j\varphi}\}\rangle N \qquad (12)$$

where $\langle \dots \rangle$ stands for exception value. By using an increasing number of necessary unwrapped neighbours in the search for the growth pixels a bigger prediction window for low coherence it is possible get faultless result until critical coherence of 0.3. Below the limit our algorithm is not stable and local bounded errors may occur. Unwrapped phase value in these region whose coherence coefficient is less than 0.3 can be obtained using Least-Squares method.

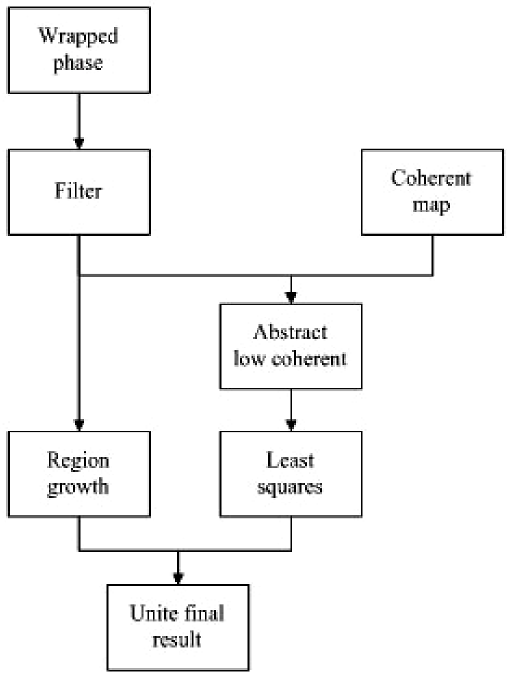

Fig.2 Flow chart of combination of Region-Growth and Least-Squares

4. Analyses of the Experimental result

The Fig. 3 as experimental data is an interference figure that is obtained from two single Look Complex images in the Three Gorges area on January 20th, 1996 and January 21st, 1996. It is used to confirms this new phase unwrapping method which was proposed in this paper. Data quality in the image's partial region is better. We can see the obvious and clear interference stripe. But from the coherent map Fig 4 we also find that there are a mass of noise in the regions near central section and lower section of the Fig 3 and image quality in the regions is very bad. So there are a lot of 'isolated island', that to say regions that can't be unwrapped, in Fig 5. There are no regions that can't be unwrapped in Fig 6, however in contrast with Fig 5 it lose many details of phase of interference. It shows that Fig 5 is less accurate than Fig 6. The Fig 7 is the final phase unwrapping result. We can find that not only there are no 'isolate island' in it but also it don't lose details of the final result.

5. Conclusion

This paper proposes a new phase unwrapping method that integrates merits of Path-Integral method and Least-Squares method. It can reconstruct wrapping phase of interference image, even low coherence or high-noise one. The final result is very accurate. Though it takes more time to unwrap phase using the method, it is worth.

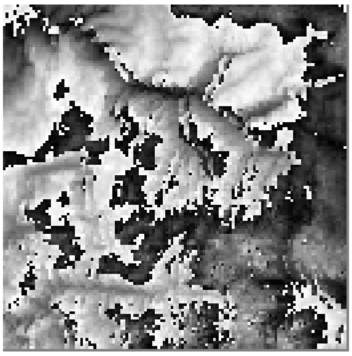

Fig.3

Fig.4

Fig.5

Fig.6

Fig. 7

Fig.3 Phase of interference before unwrapping

Fig.4 Coherence map

Fig.5 Result of Region-Growth method

Fig.6 Result of Least-Squares method

Fig. 7 Result of conbined method

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