A Frequency Domain GSC Algorithm Based on Wavelet Filter

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Abstract – *This paper proposes D_4 FLMS-GSC algorithm interpreted as Daubechies D_4 wavelet filter instead of subtractor filter which processes array antenna output. The structure of the proposed D_4 FLMS-GSC algorithm has characteristic of reducing the computational requirement one-half compared to the FLMS-GSC algorithm. In addition, we obtain the MSE characteristics and adaptive beampattern of the D_4 FLMS-GSC structure, and compared with the performance of FLMS-GSC algorithm. The D_4 FLMS-GSC algorithm proposed in this paper shows less computational requirement and gives the better performance, compared to the FLMS-GSC structure suggested by Chen-Fang.*

Keywords:. Wavelet Filter, GSC, FLMS-GSC, D_4 FLMS-GSC

1 Introduction

Adaptive beamformer consists of a multi-input single–output processor together with an adaptive algorithm. Frost has proposed the 'constrained Least-Mean-Square (LMS)' algorithm[1]. The algorithm attempts to minimize the noise power at the array output while maintaining a chosen frequency response in the direction of interest. Griffiths and Jim has proposed an alternative beamforming structure called Generalized Sidelobe Canceller (GSC) based on the constrained LMS algorithm[2]. The GSC allows the constrained LMS problem to be transformed into an unconstrained one. Although the application of adaptive LMS algorithm in time domain has the advantage of requiring less computational requirement, their convergence rate as the ratio of maximum to minimum eigenvalues of the input correlation matrix increases. Chen-Fang has proposed a new GSC structure, called FLMS-GSC, which applies adaptive algorithm after transforming the time domain data into frequency domain data. This paper proposes D_4 FLMS-GSC algorithm, and Figure 1 represents this new structure. As described in Figure 1, in the new structure, Daubechies D_4 wavelet filter is used instead of subtractor filter which processes array antenna output. We analyze the

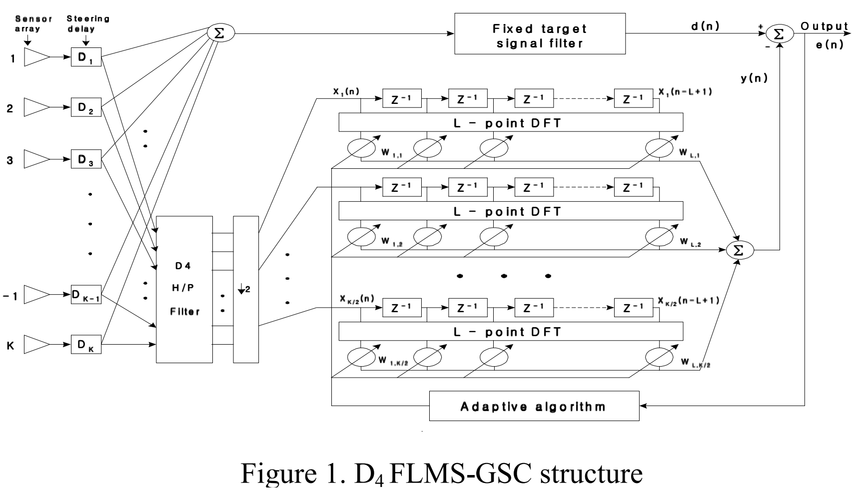


Figure 1. D_4 FLMS-GSC structure

characteristics of the proposed D_4 FLMS-GSC structure. In addition, we obtain the MSE characteristics and adaptive beampattern of the D_4 FLMS-GSC structure, and compared with the performance of FLMS-GSC algorithm suggested by Chen-Fang.

2 GSC Algorithm

The GSC algorithm make main-channel and sub-channel signal by combining the linear array antenna output properly, and remove the jamming signals by applying the adaptive LMS algorithm. In the LMS-GSC algorithm, they achieved a specific beamforming direction by using delay elements. In this structure, the output consists of two signals, $d(n)$ and $y(n)$, where $d(n)$ is obtained by summing each delay element's output and filtering this signal, and $y(n)$ is obtained by subtracting two adjacent delay element's output and applying adaptive system. The output of delay element's of target signal component of main beam direction has the same phase. As a result, the difference of two adjacent sensor element's output, $X(n)$, is regarded as jamming signal in which target signal is removed. Each subtractor's output, $X(n)$, is applied to (L-1) TDL(Tapped Delay Line), and is processed based on adaptive LMS algorithm. The $y(n)$ is the summation result of these signals.

Although the application of adaptive LMS algorithm in time domain has the advantage of requiring less computational requirement, this algorithm shows poor convergence characteristics. To improve the convergence characteristics, Chen and Fang[3], and An and Champagne[4] have proposed different realizations of the GSC using the frequency domain LMS algorithm, called FLMS-GSC and NDLMS-GSC. However these approaches have disadvantage of increased computational requirement due to transformation to frequency domain.

3 D_4 FLMS-GSC Algorithm

In this paper, we propose a new GSC structure, called D_4 FLMS-GSC (Daubechies D_4 wavelet filter based FLMS-GSC). In the newly proposed structure, we apply wavelet filter to array output signal instead of using subtracter in the Chen-Fang's FLMS-GSC. Figure 1 represents Daubechies D_4 wavelet filter based GSC structure. As described in Figure 1, the new GSC use Daubechies D_4 wavelet high pass filter to process sensor output, instead of subtraction module in the Chen-Fang's FLMS-GSC. In Figure 1, the error signal $e(n)$ can be minimized by applying the adaptive LMS scheme iteratively. In this structure, we applied Daubechies D_4 wavelet filter described in equation (1), (2)[4].

Lowpass filter :

$$H(z)=\frac{1}{4}\left[(1+\sqrt{3})+(3+\sqrt{3})z^{-1}+(3+\sqrt{3})z^{-2}+(1-\sqrt{3})z^{-3}\right] \quad (1)$$

Highpass filter:

$$G(z)=\frac{1}{4}\left[(1-\sqrt{3})-(3-\sqrt{3})z^{-1}+(3+\sqrt{3})z^{-2}-(1+\sqrt{3})z^{-3}\right] \quad (2)$$

In Figure 1, the desired signal, $d(n)$, is obtained by summing the downsampled signal of lowpass filter output., and is identical with the desired signal of conventional GSC structures. Since the reference signal, $x_i(n)$, is the downsampled signal of highpass filter output, the number of TDL is reduced compared with the conventional GSC structures. Analysis shows that the number of complex multiplication required is reduced to one half compared to Chen-Fang's FLMS-GSC structure. In Table 1, we summarized the number of complex multiplication of two schemes.

Table 1. Number of complex multiplication per iteration of two GSC algorithms

Algorithm	**FLMS-GSC**	**D4 FLMS-GSC**
Complex Multiplication/ Iteration	$2(K_1K_2-1)L+(K_1K_2-1)L\log_2 L$	$K_1K_2(L+1)+\frac{K_1K_2}{2}L\log_2 L$

To make mathematical analysis, let $u_{ij}(t)$ be the output of $(K_1 \times K_2)$ planar antenna array and $u_k(t)$ be the transformed signal to one dimension.

$$u_k(t)=u_{ij}(t), k=jK_1+i, 0\le i\le K_1, 0\le j\le K_2 \quad (3)$$

In equation (3), a linear array that has K_1 elements can be represented as a ($K_1 \times 1$) planar antenna array. In this signal, the high frequency component that comes from Daubechies D_4 highpass filter can be represented as equation (4).

$$x_j(t) = \sum_{x=0}^{3} u_{2j+k}(t) g(t), u_{K_1 K_2+1}(t) = u_1(t) \quad (4)$$

In equation (4), $g(k)$ represents coefficients of Daubechies D_4 highpass filter. Let $x_j(n)$ be the discrete signal corresponding to j th jamming signal $x_j(j)$. In this case, we can represent the reference signal at time t=n as vector $X(n)$, as described in equation (5).

$$X(n) = \left[X_0^T(n), X_1^T(n), \cdots X_{K_1 K_2/2-1}^T(n) \right]^T,$$

$$X(n) = \left[x_j(n), x_j(n-1), \cdots x_j(n-L+1) \right]^T \quad (5)$$

$$j = 0, 1, \cdots K_1 K_2 / 2 - 1$$

In equation (5), X_j represents j th TDL vector. By applying weighting vector $W(n)$ to jamming signal vector $X(n)$ the signal is processed based on adaptive LMS algorithm. Equation (6) represents the weighting vector.

$$W(n) = \left[W_0^T(n), W_1^T(n), \cdots, W_{K_1 K_2/2-1}^T) \right]^T$$

$$W(n) = \left[w_j(n), w_j(n-1), \cdots, w_j(n-L+1) \right]^T \quad (6)$$

$$j = 0, 1, \cdots, K_1 K_2 / 2 - 1$$

In this case, let $y(n)$ be the summation of TDL output in which weighting vector is applied, $y(n)$ is represented as equation (7).

$$y(n) = W^H X(n) \quad (7)$$

In equation (7), H represents Hermite operation. To obtain optimal weighting vector by minimizing square of $e(n) = d(n) - y(n)$, we update the weighting vector iteratively by applying LMS algorithm. Equation (8) represents the relationship to update weighting vector iteratively.

$$W(n+1) = W(n) + 2_{\mu} X(n) e^*(n) \quad (8)$$

In equation (8), μ represents the step size to control the convergence speed and stability.

4 Simulation

Based on simulation, we compared the performance of new D_4 FLMS-GSC with Chen-Fang's FLMS-GSC. We obtain the MSE characteristics and adaptive beampattern of the D_4 FLMS-GSC structure, and compared with the performance of FLMS-GSC algorithm suggested by Chen-Fang.

The simulation environments are as follows;

- Number of sensor elements K = 16
- Tab length L = 8
- Target signal: frequency = 0.1 (normalized), incident angle = 0, SNR = 10dB
- Jamming signal 1: frequency = 0.3 (normalized), incident angle = 34, JNR = 20dB
- Jamming signal 2: frequency = 0.4 (normalized), incident angle = -49, JNR = 40dB
- Jamming signal 3: frequency = 0.25 (normalized), incident angle = -24, JNR = 30dB

Figure 2 represents received target signal. As described in Figure 2, the new D_4 FLMS-GSC structure shows good target signal characteristics which can be applicable to beamforming system.

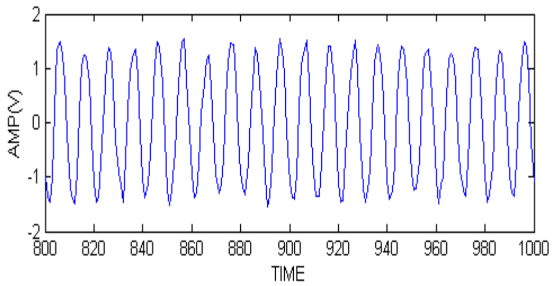
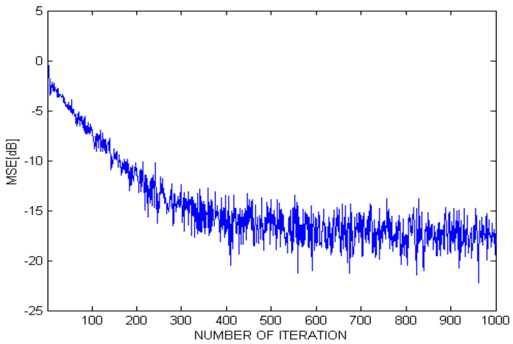
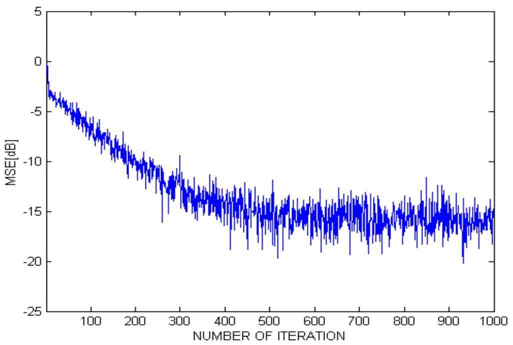


Figure 2. Received taregt signal characteristics of D_4 FLMS-GSC structure

Figure 3 represents Mean Square Error (MSE) characteristics of two GSC algorithms. As described in Figure 3, the convergence characteristics of the new D_4 FLMS-GSC is almost the same as Chen-Fang's FLMS-GSC, although the computational requirement is reduced to one half.



(a) Chen-Fang's FLMS-GSC



(b) D_4 FLMS-GSC

Figure 3. MSE characteristics of two GSC algorithms

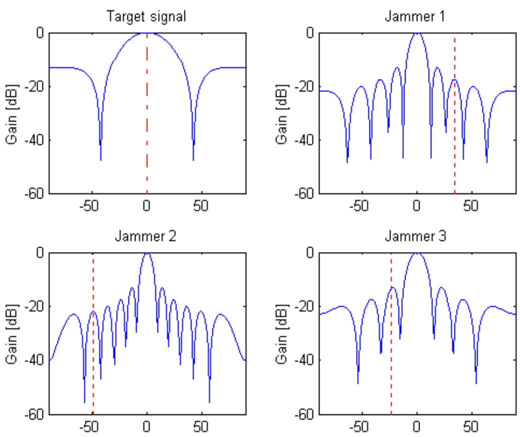


Figure 4. Beam pattern of target and jamming signals before application of D_4 LMS_GSC algorithm

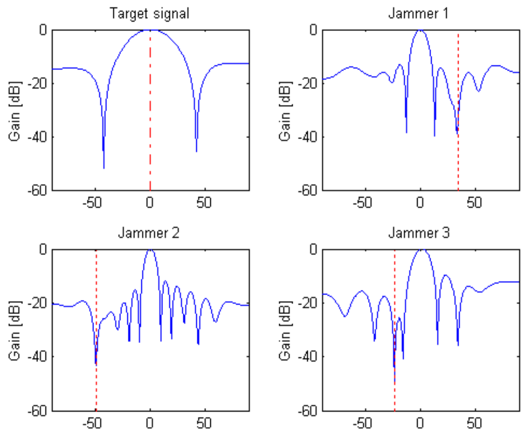


Figure 5. Beam pattern of target and jamming signals using the D_4 LMS_GSC algorithm after 1000 iterations

Figure 4, 5 represents the jamming signal nullifying characteristics of D_4 LMS_GSC algorithms. As described in Figure 5, after the application of the new D_4 LMS-GSC algorithm, the sidelobe level is reduced significantly (more than 45dB) in the three jamming signal directions. after 1000 iterations. In addition, D_4 LMS-GSC algorithm shows better jamming signal nullifying characteristics in the direction of –49 and –24, compared to FLMS-GSC algorithm.

5. Conclusions

This paper proposes D_4 FLMS-GSC algorithm interpreted as Daubechies D_4 wavelet filter instead of subtracter filter which processes array antenna output. The structure of the proposed D_4 FLMS-GSC algorithm has characteristic of reducing the computational requirement one-half compared to the FLMS-GSC algorithm suggested by Chen-Fang. In addition, we obtain MSE characteristics and adaptive beampattern of the D_4 FLMS-GSC structure, and compared with the performance of FLMS-GSC algorithm. The simulation result show that the D_4 FLMS-GSC algorithm proposed in this paper shows better or almost the same performance, compared to the FLMS-GSC structure suggested by Chen-Fang. As a result, the newly proposed structure gives more chance for real time implementation of GSC algorithm.

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