Non-Equidistance Grey Model Based on Grey Interval Weighting Accumulated Generating Operation

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Abstract- *Non-equidistance grey model NGM(1,1) which was proposed by Deng has emerged as a powerful tool for the prediction and analysis of non-equidistance data series under uncertain system. It carries out the system analysis and prediction based on accumulated generating operation (AGO) to reduce the randomness of original series and obtain high accuracy. But the AGO only used observed whitening data, thus it disregards the existences of grey information in the grey interval. Such that the accuracy of NGM(1,1) is not good and predicted value will bring large error. In this paper, we proposed a new NGM(1,1) based on grey interval weighting accumulated generating operation (GIWAGO) to enhance the predicted accuracy. The work procedure is shown as follows briefly: First, we presented a whitening coefficient based on grey interval to generate weighting accumulated series. Second, after the whitening coefficient was set initial value, we presented Taylor approximation method to optimize the whitening coefficient and made predicted error reduce to minimum. Finally, We presented three examples to verify the effectiveness of proposed model.*

Keywords: NGM(1,1), GIWAGO, rough range, optimization method, Taylor approximation method

1 Introduction

In the research field of system analysis, correct applications of the mathematical analysis and the evaluation method by obtained information are very important. However the behaviors of most systems are uncertain and the effects of other systems on the system being monitored are also unclear. Therefore there is often difficult to obtain the accurate data of system. Such a state is view as a poor state of information. Thus, it is difficult to predict the development of system. Up to present, proposed methods for prediction problem can be roughly divided into four types: time series method, regression method, expert-based method and neural network based method. In order to obtain high accuracies, these methods are very complex and need a large amount of sample data. Thus, these methods are limited by the used history data [1]. Therefore, they are often difficult to carry on and even not approachable due to cost consideration [2]. In additon, when sample data are not only few but also random, their prediction accuracies are low [3].

In 1982, Deng proposed grey system theory [4] to study uncertainty of system. It avoids the inherent defects of conventional, large sample statistical methods, and only requires a limited amount of discrete data to estimate the behavior of a system with incomplete information. Non-equidistance grey model NGM(1,1) [5]-[6] which stand for a single variable first order differential equation prediction model based on grey system theory has already been proposed to predict and analyze the non-equidistance data series from uncertain system. NGM(1,1) carries out the system analysis and prediction based on accumulated generating operation (AGO) of original data series. The AGO converts a series lacking and obvious regularity into a strictly monotonically increasing series to reduce the randomness of the series, increase the smoothness of the series, and minimize interference from the random information. Up to present, NGM(1,1) is widely used in many fields which only can obtain the sample data of non-equidistance series, such as mechanical engineering [7]-[12], natural science [13],[14] etc.. However, the AGO only used observed whitening data, thus it disregards the existences of grey information in the grey interval [3]. Such that the predicted accuracy is not satisfactory. In some application fields which the demands of accuracies are high, it is impossible to use conventional NGM(1,1), therefore the application range of grey system theory is limited.

In this paper, we proposed a new NGM(1,1) based on grey interval weighting accumulated generating operation (GIWAGO) to enhance the predicted accuracy. The work procedure is shown as follows briefly: First, we presented a whitening coefficient based on grey interval to generate weighting accumulated data series. Second, after whitening coefficient was set initial value, we presented Taylor approximation method to optimize the whitening coefficient and made predicted error reduce to minimum. Finally, We presented three examples to verify proposed model. The experimental results show that proposed model has good performance.

This paper is organized as follows: Section 2 describes grey system theory which include some basic definitions, grey 1-AGO and 1-IAGO, and NGM(1,1) model. Section 3 introduces proposed NGM(1,1) model based on GIWAGO. In Section 4, we introduce the optimization method of whitening coefficient by Taylor approximation method. In Section 5, we present three application examples to verify the proposed NGM(1,1) model. Finally, conclusions are described in Section 6.

2 Grey system theory

2.1 Basic definitions

In recent years, grey system theory has become a very effective method of solving uncertainty problems under discrete data and incomplete information. Grey system theory has now been applied to various areas such as forecasting, system control, decision making and computer graphics. Here, we give some basic definitions of grey system, grey interval, grey number and whitening method in grey system theory.

Definition 1 A grey system [4]-[6] is defined as a system containing uncertain information presented by grey interval numbers and grey variables.

Definition 2 In grey system, when a prediction model uses an observed data set, there will be a numerical interval accompanying it. This numerical interval will contain the accuracy and the other sources of uncertainty that are associated with the observed values in the data set. The numerical interval is defined as grey interval.

Definition 3 The number of grey interval is defined as grey number. Grey number means that the certain value is unknown, but the rough range is known. Grey interval can be taken as a special grey number $\otimes X_g$, with bound values X_d and X_u:

$$\otimes X_g = [X_d, X_u] \tag{1}$$

where X_d is the lower limit and X_u is upper limit.

Definition 4 The whitening method of grey number is given as

$$X_g = (1-\lambda)X_d + \lambda X_u \tag{2}$$

where $\lambda \in [0,1]$, λ is called whitening coefficient.

2.2 Grey 1-AGO and 1-IAGO

The most critical feature of NGM(1,1) is the use of grey generating approaches to reduce the variation of the original data series by transforming the data series linearly. The most commonly seen and applied grey generating approaches are the accumulative generating operation (AGO) and the inverse accumulative generating operation (IAGO). The AGO converts a series lacking any obvious regularity into a strictly monotonically increasing series to reduce the randomness of the series, increase the smoothness of the series, and minimize interference from the random information.

Definition 5 Assume that $x^{(0)} = \{x^{(0)}(t_1), x^{(0)}(t_2), \cdots, x^{(0)}(t_n)\}$ is original series of real numbers with irregular distribution. Where $x^{(0)}(t_i)$ is the system output data at time t_i. If sampling interval $\Delta t_i = t_i - t_{i-1} \neq$ constant, then $x^{(0)}$ is called a non-equidistance series.

Then $x^{(1)}$ is viewed as 1-AGO generation series for $x^{(0)}$, if $\forall x^{(1)}(t_j) \in x^{(1)}$ can satisfy

$$x^{(1)}(t_j) = \sum_{\substack{i=1 \\ x^{(0)}(t_i)\,\in\, x^{(0)}}}^{j} x^{(0)}(t_i) \tag{3}$$

Then $x^{(1)} = \{\sum_{i=1}^{1} x^{(0)}(t_i), \sum_{i=1}^{2} x^{(0)}(t_i), \cdots, \sum_{i=1}^{n} x^{(0)}(t_i)\}$, which is the first order AGO series obtained from $x^{(0)}$.

Definition 6 From Eq. (3), it is obvious that the original data $x^{(0)}(t_i)$ can be easily recovered from $x^{(1)}(t_i)$ as

$$x^{(0)}(t_i) = x^{(1)}(t_i) - x^{(1)}(t_{i-1}) \tag{4}$$

where $x^{(0)}(t_1) = x^{(1)}(t_1), x^{(1)}(t_i) \in x^{(1)}$. This operation is called first order IAGO.

2.3 NGM(1,1) model

Definition 7 If we have $n \geq 4$, $x^{(0)}, x^{(1)} \in R^+$, and can satisfy the precondition:

$$\left.\begin{array}{l} \sigma^{(1)}(t_i) \in (e^{-\frac{2}{n+1}}, e^{+\frac{2}{n+1}}) \\ \sigma^{(1)}(t_i) = \frac{x^{(1)}(t_{i-1})}{x^{(1)}(t_i)} \end{array}\right\} \tag{5}$$

where $\sigma^{(1)}(t_i)$ is called class ratio.

The non-equidistance grey prediction NGM(1,1) model can be expressed by one variable, and first order differential equation.

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = b \tag{6}$$

The solution for Eq. (6) will be obtained by Laplace transform [15] as the following:

By Laplace transform, Eq. (6) can be expressed in frequency domain as

$$sx^{(1)}(s) - u(0) + ax^{(1)}(s) = \frac{b}{s} \tag{7}$$

where $u(0)$ is intial value of system at time 0. $u(0) = x^{(0)}(t_1) = x^{(1)}(t_1)$.

$$x^{(1)}(s) = \frac{u(0) - \frac{b}{a}}{s+a} + \frac{\frac{b}{a}}{s} \tag{8}$$

By Laplace inversion transform, the solution of discrete system form are obtain by

$$\hat{x}^{(1)}(t_i) = \Big(x^{(0)}(t_1) - \frac{b}{a}\Big)e^{-a(t_i - t_1)} + \frac{b}{a} \tag{9}$$

where the coefficients a and b are called developing and grey input coefficient, respectively.

By least-square method, they can be obtained as

$$\begin{bmatrix} a \\ b \end{bmatrix} = \big(A^T A\big)^{-1} A^T X_n \tag{10}$$

$$A = \begin{bmatrix} \frac{1}{2}\big(x^{(1)}(t_1) + x^{(1)}(t_2)\big)\Delta t_2 & \Delta t_2 \\ \frac{1}{2}\big(x^{(1)}(t_2) + x^{(1)}(t_3)\big)\Delta t_3 & \Delta t_3 \\ \vdots & \vdots \\ \frac{1}{2}\big(x^{(1)}(t_{n-1}) + x^{(1)}(t_n)\big)\Delta t_n & \Delta t_n \end{bmatrix} \tag{11}$$

$$X_n = \begin{bmatrix} x^{(0)}(t_2) \\ x^{(0)}(t_3) \\ \vdots \\ x^{(0)}(t_n) \end{bmatrix} \tag{12}$$

where $\Delta t_i = t_i - t_{i-1}$.

By 1-IAGO, the predicted equation is,

$$\hat{x}^{(0)}(t_i) \quad = \quad \hat{x}^{(1)}(t_i) - \hat{x}^{(1)}(t_{i-1}) \tag{13}$$

where $\hat{x}^{(0)}(t_1) = x^{(0)}(t_1)$.

From Eq. (13), the data series $\{\hat{x}^{(0)}(t_1), \hat{x}^{(0)}(t_2), \cdots, \hat{x}^{(0)}(t_{n+k})\}$, $k \geq 1$ are called predicted data series.

The estimation error is given by the root mean square percentage error (RMSPE) [16].

$$\text{minimize:RMSPE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} \frac{e^2(t_i)}{[x^{(0)}(t_i)]^2}} \times 100\% \tag{14}$$

$$e(t_i) = x^{(0)}(t_i) - \hat{x}^{(0)}(t_i) \tag{15}$$

where $\hat{x}^{(0)}(t_i)$ is the predicted value for time t_i.

3 Proposed NGM(1,1) Model

From Definition 5, we know original non-equidistance series $x^{(0)} = \{x^{(0)}(t_1), x^{(0)}(t_2), \cdots, x^{(0)}(t_n)\}$ is observed whitening data series. The conventional AGO only used these observed whitening data to establish grey differential equation NGM(1,1) model. We think that each interval of observed whitening data consists of uncertain grey information. These intervals can be viewed grey interval [3]. If only used observed whitening data, thus it disregards the existences of grey information in the grey interval. As a result, the predicted accuracy becomes low. We proposed a new NGM(1,1) based on grey interval weighting accumulated generating operation (GIWAGO) to enhance the predicted accuracy. The predicted value is obtained by grey interval weighting inverse accumulated generating operation (GIWIAGO). The proposed model is described as follows.

3.1 1-GIWAGO and 1-GIWIAGO

Theorem 1 For non-equidistance original whitening series $x^{(0)} = \{x^{(0)}(t_1), x^{(0)}(t_2), \cdots, x^{(0)}(t_n)\}$, then $\otimes x^{(0)} = \{\otimes x^{(0)}(t_1), \otimes x^{(0)}(t_2), \cdots, \otimes x^{(0)}(t_n)\}$ is the grey interval generation series for $x^{(0)}$. $\otimes x^{(1)}$ is viewed as 1-GIWAGO generation series for $x^{(0)}$. if $\forall \otimes x^{(1)}(t_j) \in \otimes x^{(1)}$ can satisfy

$$\otimes x^{(1)}(t_j) = \sum_{i=1}^{j} \big(\otimes x^{(0)}(t_i)\big)\Delta t_i \tag{16}$$

where

$$\otimes x^{(0)}(t_i) = (1-\lambda)x^{(0)}(t_{i-1}) + \lambda x^{(0)}(t_i) \tag{17}$$

$\lambda \in [0,1]$, λ is called whitening coefficient; $x^{(1)}(t_1) = x^{(0)}(t_1)$.

Then $\otimes x^{(1)} = \{\otimes x^{(1)}(t_1), \otimes x^{(1)}(t_2), \cdots, \otimes x^{(1)}(t_n)\}$, which is the first order GIWAGO series obtained from $x^{(0)}$.

Proof: We know that the $n-1$ intervals $[x^{(0)}(t_{i-1}), x^{(0)}(t_i)]$, $(i = 2, 3, \cdots, n)$ consists of uncertain grey information. They can be viewed as $n-1$ grey intervals shown in Eq. (1). According to Eq. (2), we can obtain their whitening method. In addition, since $x^{(0)}$ is non-equidistance original series, we can use the AGO of weighting $\Delta t_i = t_i - t_{i-1}$ to reduce the randomness of original series further. □

Definition 8 From Eq. (16), it is obvious that the original interval data $\otimes x^{(0)}(t_i)$ can be easily recovered from $\otimes x^{(1)}(t_i)$ as

$$\otimes x^{(0)}(t_i) = \frac{\otimes x^{(1)}(t_i) - \otimes x^{(1)}(t_{i-1})}{\Delta t_i} \tag{18}$$

where $\otimes x^{(1)}(t_i) \in \otimes x^{(1)}$. This operation is called first order GIWIAGO.

3.2 Proposed NGM(1,1) model

According to 1-GIWAGO and 1-GIWIAGO, the proposed NGM(1,1) model can be calculated by below theorem.

Theorem 2 For non-equidistance grey differential equation NGM(1,1) model shown in Eq. (6), the solution equation can be calculated by Eq. (10). The coefficients a and b of Eq. (10) can be calculated by

$$\begin{bmatrix} a \\ b \end{bmatrix} = \left(A_p^T A_p\right)^{-1} A_p^T X_n \tag{19}$$

where

$$A_p = \begin{bmatrix} \frac{1}{2}\left(\otimes x^{(1)}(t_1) + \otimes x^{(1)}(t_2)\right) & 1 \\ \frac{1}{2}\left(\otimes x^{(1)}(t_2) + \otimes x^{(1)}(t_3)\right) & 1 \\ \vdots & \vdots \\ \frac{1}{2}\left(\otimes x^{(1)}(t_{n-1}) + \otimes x^{(1)}(t_n)\right) & 1 \end{bmatrix} \tag{20}$$

$$X_n = \begin{bmatrix} \otimes x^{(0)}(t_2) \\ \otimes x^{(0)}(t_3) \\ \vdots \\ \otimes x^{(0)}(t_n) \end{bmatrix} \tag{21}$$

$\otimes x^{(1)}(t_j) = \sum_{i=1}^{j}(\otimes x^{(0)}(t_i))\Delta t_i$ shown in Eq. (16); $\otimes x^{(0)}(t_i) = (1-\lambda)x^{(0)}(t_{i-1}) + \lambda x^{(0)}(t_i)$ shown in Eq. (17).

Then by 1-GIWIAGO shown in Eq. (18), the predicted interval series, $\{\otimes\hat{x}^{(0)}(t_1), \otimes\hat{x}^{(0)}(t_2), \cdots, \otimes\hat{x}^{(0)}(t_{n+k})\}$, $k \geq 1$ can be obtained. The mathematical model of the differential equation shown by abovementioned Eq. (6) which can be calculated by Eq. (19) is defined as proposed NGM(1,1) model.

Proof: Eq. (16) and Eq. (17) are substituted for Eq. (6), we can obtain

$$\otimes x^{(0)}(t_i) + a \otimes x^{(1)} = b \tag{22}$$

Then by least-square method, we can obtain the calculation method of Eq. (19). Finally, by 1-GIWIAGO shown in Eq. (18), the predicted interval series, $\{\otimes\hat{x}^{(0)}(t_1), \otimes\hat{x}^{(0)}(t_2), \cdots, \otimes\hat{x}^{(0)}(t_{n+k})\}$, $k \geq 1$ can be obtained. □

When λ is set initial value, we can obtain the whitening precited series $\{\hat{x}^{(0)}(t_1), \hat{x}^{(0)}(t_2), \cdots, \hat{x}^{(0)}(t_{n+k})\}, k \geq 1$ for $x^{(0)}$.

4 Optimization method by Taylor approximation method

Theorem 3 After whitening coefficient λ of proposed NGM(1,1) shown in Theorem 2 is set initial value, then the initial value can as the initial parameter of Taylor approximation method and can be adjusted repeatedly until reaches the optimal values and make the predicted error reduce to the minimum.

The optimization process of whitening coefficient λ is described as follows.

Step 1: Initialization

(a) Setting updated times K=0.

(b) Setting initial coefficient $\lambda = 0.5$.

(c) Setting objective function vector G:

$$G = \left[x^{(0)}(t_1), x^{(0)}(t_2), \cdots, x^{(0)}(t_n)\right]^T \tag{23}$$

where $x^{(0)}(t_i) \in G, \{x^{(0)}(t_i), i = 1, 2, \cdots, n\}$ is non-equidistance original series

(d) Setting approximated function vector $F^{(K)}$:

$$F^{(K)} = \left[\hat{x}^{(0)(K)}(t_1), \hat{x}^{(0)(K)}(t_2), \cdots, \hat{x}^{(0)(K)}(t_n)\right]^T \tag{24}$$

where $\hat{x}^{(0)(K)}(t_i) \in F^{(K)}$, $\{\hat{x}^{(0)(K)}(t_i), i = 1, 2, \cdots, n\}$ is K times generated predicted series of proposed non-equidistance GM(1,1) model. When K=0, $F^{(0)}$ is the initial predicted series $\hat{x}^{(0)}$.

(e) Setting approximated parameters $\lambda^{(K)}$:

where $\lambda^{(K)}$ is K times generated parameter. When $K = 0$, $\lambda^{(0)} = 0.5$ become initial parameter.

Step 2: Calculation of approximated function vector $F^{(K)}$ according to first Taylor development

$$F^{(K+1)} = F^{(K)} + F_1^{(K)}(\lambda^{(K+1)} - \lambda^{(K)}) \tag{25}$$

where $F_1^{(K)} = \frac{\partial F^{(K)}}{\partial \lambda^{(K)}}$.

Step 3: Setting evaluation function $Q^{(K)}$

$$Q^{(K)} = \left[F_D^{(K)} - F_1^{(K)}\eta_1^{(K)}\right]^T \left[F_D^{(K)} - F_1^{(K)}\eta_1^{(K)}\right] \tag{26}$$

where $F_D^{(K)} = G - F^{(K)}, \eta_1^{(K)} = \lambda^{(K+1)} - \lambda^{(K)}$.

Step 4: Detect stop criterion

If $Q^{(K)} \leq \varepsilon$, stop; otherwise, go to Step 5.

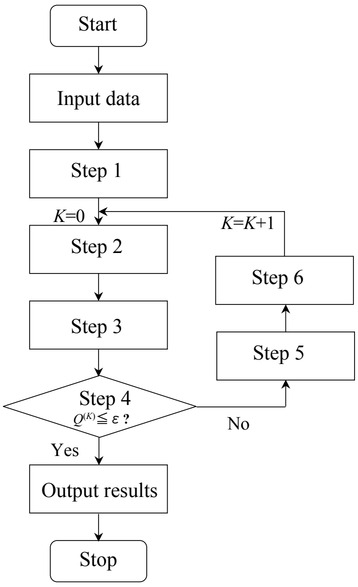

Figure 1: The flow chart of optimization process of λ by Taylor approximation method

where ε is allowable error.

Step 5: Update approximated parameter $\lambda^{(K)}$

$$\lambda^{(K+1)} = \lambda^{(K)} + \left[A^{(K)T}A^{(K)}\right]^{-1} A^{(K)T} F_D^{(K)} \quad (27)$$

where $A^{(K)} = F_1^{(K)}$.

Step 6: Increase updated times K=K+1; go to Step 2.

The parameter $\lambda^{(K)}$ is approximated for K times. At last, when $Q^{(K)} \leq \epsilon$, we can obtain the solution of the approximation calculation which is $\lambda^{(K)}$. Vector $F^{(K)}$ at this time become the predicted generation series $\hat{x}^{(0)}(t_i)$ as the result of approximation calculation. The flow chart is shown in Fig. 1.

Proof: Taylor approximation method [17],[18] which combines the Taylor development with the least squares method is an approximate calculation method of multi-times to obtain the optimal parameter and makes the convergent error reduce to the minimum. Therefore, the whitening coefficient λ is set initial value, then the initial value can be optimized by Taylor approximation method. □

5 Applications and analysis

We presented three application examples to verify the effectiveness of proposed NGM(1,1) model. And its predicted accuracy is compared with conventional NGM(1,1) model. The improved rate σ[%] is given as

$$\sigma[\%] = \frac{\mathrm{RMSPE}\big(\mathrm{NGM(1,1)}\big) - \mathrm{RMSPE}\big(\mathrm{Proposed}\big)}{\mathrm{RMSPE}\big(\mathrm{NGM(1,1)}\big)} \times 100\% \quad (28)$$

where RMSPE is calculated by Eq. (14).

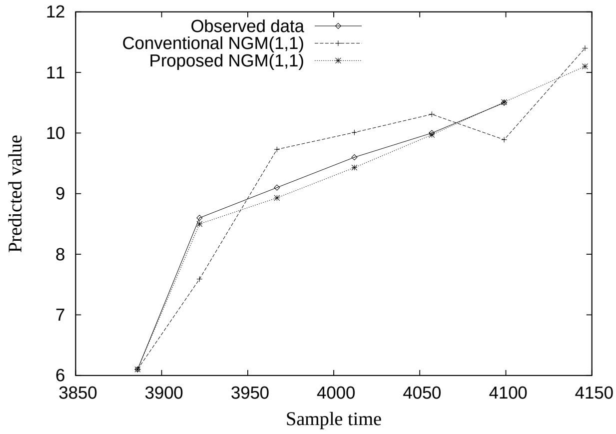

Figure 2: Predicted results for example 1

Table 1: Predicted results for example 1

Time	Observed data	NGM(1,1) model	Pro. model
3886	6.1	6.10	6.10
3922	8.6	7.59	8.50
3967	9.1	9.73	8.93
4012	9.6	10.01	9.43
4057	10.0	10.31	9.97
4099	10.5	9.89	10.51
4146	N/A	11.38	11.10
RMSPE [%]		15.77	**2.80**
Improved rate σ [%]		–	**82.23**

5.1 Example 1

The example is taken from [7], the original observed data are listed in Table 1. We used original sample data shown in Table 1 to establish NGM(1,1). The predicted results from NGM(1,1) model is also shown in Table 1, and it is plotted in Fig. 2. As we mentioned before, the predicted curve cannot keep track of the actual one. For proposed NGM(1,1) model, when initial whitening coefficient λ is set to 0.5 and through K times undated calculation by Taylor approximation method, the evaluation function $Q^{(K)}$ shown in Eq.

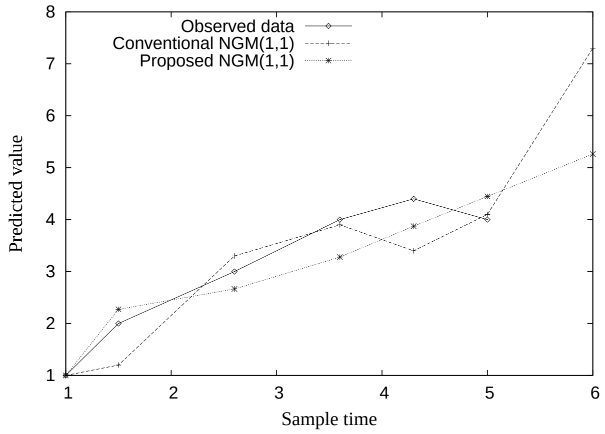

Figure 3: Predicted results for example 2

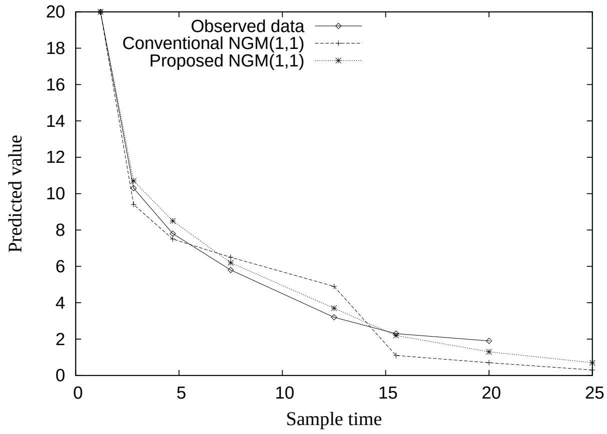

Figure 4: Predicted results for example 3

Table 2: Predicted results for example 2

Time	Observed data	NGM(1,1) model	Pro. model
1.0	1.0	1.00	1.00
1.5	2.0	1.23	2.27
2.6	3.0	3.32	2.67
3.6	4.0	3.95	3.28
4.3	4.4	3.43	3.87
5.0	4.0	4.10	4.45
6.0	*N/A*	7.28	5.26
RMSPE [%]		45.84	**30.07**
Improved rate σ [%]		–	**31.41**

Table 3: Predicted results for example 3

Time	Observed data	NGM(1,1) model	Pro. model
1.2	20.0	20.00	20.00
2.8	10.3	9.35	10.70
4.7	7.8	7.45	8.48
7.5	5.8	6.47	6.21
12.5	3.2	4.91	3.74
15.5	2.3	1.14	2.16
20.0	1.9	0.74	1.32
25.0	*N/A*	0.28	0.70
RMSPE [%]		96.76	**37.17**
Improved rate σ [%]		–	**61.58**

(26) will converge at the end. This time, $\lambda = 0.90$ is obtained and evaluation function $Q^{(K)}$ becomes allowable error ϵ of system. The predicted results of proposed NGM(1,1) are also shown in Table 1 and Fig. 2. It is obvious that the extreme-effect has been somewhat removed and the predicted curve is on the right track. Finally, the accuracy comparison and improved rate σ[%] of proposed NGM(1,1) are also listed in Table 1.

5.2 Example 2

Let us consider another example presented by [19], the original observed data are listed in Table 2. The same calculation method to example 1, For proposed NGM(1,1) model, through K times undated calculation of whitening coefficient λ by Taylor approximation method, $\lambda = 0.76$ is obtained. The predicted results of two models of conventional NGM(1,1) and proposed NGM(1,1) are listed in Table 2 and Fig. 3. Finally, the accuracy comparison and improved rate σ[%] of proposed NGM(1,1) are also listed in Table 2.

5.3 Example 3

As a final example, let us consider the example presented by [20], the original observed data are listed in Table 3. The same calculation method to example 1, $\lambda = 0.90$ is obtained. The predicted results of two models of conventional NGM(1,1) and proposed NGM(1,1) are also listed in Table 3 and Fig. 4. Finally, the accuracy comparison and improved rate σ[%] of proposed NGM(1,1) are also listed in Table 3.

5.4 Consideration

Prediction is through obtaining the relation of already-known data to analyze the developing tendency of system in the future. We proposed improved NGM(1,1) model to enhance the predicted accuracy of conventional NGM(1,1). About accuracy improvement and calculation amount by proposed model are enunciated as follows.

(a) Accuracy Improvement:

For non-equidistance original whitening series $x^{(0)} = \{x^{(0)}(t_1), x^{(0)}(t_2), \cdots, x^{(0)}(t_n)\}$, conventional grey differential equation NGM(1,1) model is established based on AGO of original series. However, the AGO only used observed whitening data, thus it disregards the existences of grey information in the grey interval. The grey interval of observed whitening data consists of uncertain grey information [3]. We presented GIWAGO to use the information in the grey interval and solved unreasonable problem in conventional NGM(1,1) model. In addition, we presented Taylor approximation method that is an approximation calculation method multi-times to obtain the optimal parameter (whitening coefficient λ) and the convergent error can be reduced to the minimum. From examples 1-3, the improved rates 82.23%, 31.41% and 61.58% of proposed NGM(1,1) are obtained respectively.

(b) Calculation Amount

Conventional NGM(1,1) model is applied to the uncertain system analysis from small amount non-equidistance observed data. There is a advantage of small calculation amount. For proposed NGM(1,1) mode, in case of Taylor approximation method, the initial whitening coefficient λ is updated to obtain the optimal solution coefficient. In short, the calculation of the K times is needed repeatedly in addition to the past calculation. The calculation has a possibility of increasing the calculation times until the accuracy satisfy the prediction demand of system.

6 Conclusions

The major purposed of this paper is enhance the predicted performance of NGM(1,1) model based on GIWAGO. Three application examples are used to verify the effectiveness of proposed model. The effects are achieved more than conventional NGM(1,1) model. The proposed model can help in more accurate prediction, such as weather conditions, economy changes and society information etc..

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