

Design of Stable Stabilizing Regulators Using Advanced Visualization Tools

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Abstract - The well known Diophantine equation based solution for stabilizing and pole placement technique for open-loop unstable discrete-time time-delay plants, has some limitations optimizing the regulatory properties of the closed-loop. An iterative algorithm is introduced to optimize these regulators. Advanced visualization tools can help to find the stable stabilizing controllers.

Keywords: time-delay, pole-placement, discrete-time, optimization, two-degree of freedom regulator

1. Introduction

There are two major techniques available to controller design for unstable processes [1], [2]. First of them is the well known state-feedback approach allowing to place virtually arbitrary poles, so this regulator can be considered a stabilizing one. The second one is based on the solution of a *Diophantine-equation (DE)* for finding the numerator and denominator of the stabilizing pole-placement regulator. The proper use of the *DE* needs a longer practice and special knowledge handling such numerical algebraic problems, especially to find the possible orders of the regulator, having user defined factors. The state-feedback equations are much simpler.

For open-loop stable plants the application of *Youla-parametrization (YP)* gives nice results. The paper compares the *DE*-based technique with the *YP* and investigates which properties can be exactly guaranteed and which ones can not be reached. This comparing investigation is done in the framework of a *generic two-degree of freedom (GTDOF)* scheme, which is the extension of the *YP* for *two-degree of freedom (TDOF)* problems.

2. A GTDOF Controller for Stable Linear Plants

It is not so widely known, that a proper explicit pole placement and optimization can be achieved in a *TDOF* control system without the solution of a *DE*, if the plant is open-loop stable.

The first systematic method for this approach was presented in [3], introducing the *generic two-degree*

of freedom (GTDOF) scheme, when the process is open-loop stable and it is allowed to cancel the stable process poles, which case occurs at many practical tasks. This framework and topology is based on the *Youla-parametrization (YP)* [6] providing all realizable stabilizing regulators (*ARS*) for open-loop stable plants and capable to handle the plant time-delay, too.

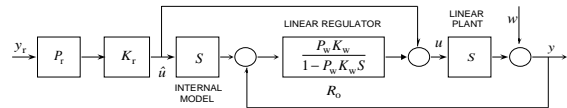


Fig. 1. The *generic TDOF (GTDOF)* control system

A *GTDOF* control system is shown in Fig. 1, where w is the output disturbance signal. The optimal *ARS* regulator of the *GTDOF* scheme can be given by an explicit form

$$R_o = \frac{P_w K_w}{1 - P_w K_w S} = \frac{Q_o}{1 - Q_o S} = \frac{P_w G_w S_+^{-1}}{1 - P_w G_w S_- z^{-d}} \quad (1)$$

where

$$Q_o = Q_w = P_w K_w = P_w G_w S_+^{-1} \quad (2)$$

is the associated optimal *Y-parameter* furthermore

$$Q_r = P_r K_r = P_r G_r S_+^{-1}; \quad K_w = G_w S_+^{-1}; \quad K_r = G_r S_+^{-1} \quad (3)$$

assuming that the process is factorable as

$$S = S_+ \bar{S}_- = S_+ S_- z^{-d} = \frac{\mathcal{B}}{\mathcal{A}} = \frac{\mathcal{B}_+ \mathcal{B}_-}{\mathcal{A}} z^{-d} \quad (4)$$

where $S_+ = \mathcal{B}_+ / \mathcal{A}$ means the *IS* and $S_- = \mathcal{B}_-$ the *IU*

factors, respectively. z^{-d} corresponds to the discrete time-delay, where d is the integer multiple of the sampling time. (In a practical case the factor S_- can incorporate the underdamped zeros and neglected poles providing realizability, too). The regulator (1) can be considered the generalization of the *Truxal-Guillemin* method [7] for stable processes. It is interesting to see how the transfer characteristics of the closed-loop look like:

$$\begin{aligned} y &= P_r K_r S y_r + (1 - P_w K_w S) w = T_r y_r + E w = \\ &= P_r G_r S_- z^{-d} y_r + (1 - P_w G_w S_- z^{-d}) w = y_t + y_d \end{aligned} \quad (5)$$

where y_t is the tracking (servo) and y_d is the regulating (or disturbance rejection) independent behaviors of the closed-loop response, respectively. So the delay z^{-d} and S_- can not be eliminated, consequently the ideal design goals P_r and P_w are biased by $G_r S_-$ and $G_w S_-$. Here P_r and P_w are assumed stable and usually strictly proper transfer functions, that are partly capable to place desired poles in the tracking and the regulatory transfer functions, furthermore they are usually referred as reference signal and output disturbance predictors. They can even be called as reference models, so reasonably $P_r(\omega=0)=1$ and $P_w(\omega=0)=1$ are selected. The unity gain of P_w ensures integral action in the regulator, which is maintained if the applied optimization provides $G_w S_-(\omega=0)=1$.

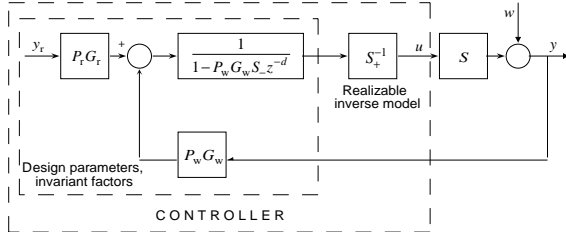


Fig. 2. Simplified form of the *GTDOF* control system

An interesting result was found in [4], that the optimization of the *GTDOF* scheme can be performed in \mathcal{H}_2 and \mathcal{H}_∞ norm spaces by the proper selection of the serial embedded filters G_r and G_w attenuating the influence of the invariant process factor S_- .

After some straightforward block manipulations the *GTDOF* control system can be transformed to a simpler form shown in Fig. 2. This form is special because the controller consists of two parts. The first part depends only on the design parameters and the invariant process factor (at a selected optimality criterion), while the second one depends only on the realizable inverse model of the plant.

3. Regulator Design by *DE* Method

Let us find the pole placement regulator in the form of a proper transfer function

$$R = \frac{\mathcal{Y}}{\mathcal{X}} \quad (6)$$

and let the required closed-loop characteristic polynomial (containing the poles to be placed) be \mathcal{P} . Assume that the denominator of the unstable process can be factorized as

$$\mathcal{A} = \mathcal{A}_+ \mathcal{A}_- \quad (7)$$

where \mathcal{A}_+ contains the stable, \mathcal{A}_- does the unstable poles, respectively. The most general *DE* allowing the cancellation of the stable polynomials \mathcal{B}_+ and \mathcal{A}_+ has the form [1]

$$\begin{aligned} (\mathcal{A}_+ \mathcal{A}_-) (\mathcal{B}_+ \mathcal{X}_d \mathcal{X}') + (\mathcal{B}_+ \mathcal{B}_- z^{-d}) (\mathcal{A}_+ \mathcal{Y}_d \mathcal{Y}') &= \mathcal{C} = \mathcal{A}_+ \mathcal{B}_+ \mathcal{P} \\ \mathcal{A} \quad \mathcal{X} \quad + \quad \mathcal{B} \quad \mathcal{Y} &= \mathcal{C} \end{aligned} \quad (8)$$

Here \mathcal{X}_d and \mathcal{Y}_d are the desired components of the numerator and denominator of the regulator to be designed. Simplifying by the cancelable factors the final reduced *DE* to be solved is given by

$$\begin{aligned} (\mathcal{A}_- \mathcal{X}_d) \mathcal{X}' + (\mathcal{B}_- z^{-d} \mathcal{Y}_d) \mathcal{Y}' &= \mathcal{P} \\ \mathcal{A}' \quad \mathcal{X}' + \quad \mathcal{B}' \quad \mathcal{Y}' &= \mathcal{P} \end{aligned} \quad (9)$$

where $\mathcal{A}' = \mathcal{A}_- \mathcal{X}_d$ and $\mathcal{B}' = \mathcal{B}_- z^{-d} \mathcal{Y}_d$ are known and the obtained regulator is

$$R = \frac{\mathcal{Y}}{\mathcal{X}} = \frac{\mathcal{A}_+ \mathcal{Y}_d \mathcal{Y}'}{\mathcal{B}_+ \mathcal{X}_d \mathcal{X}'} \quad (10)$$

It can be well seen that the obtained *stabilizing* regulator cancels only the stable zeros and stable poles, furthermore \mathcal{Y}_d and \mathcal{X}_d can really bring desired factors into the regulator transfer function. The *GTDF* regulator (1) is an integrating one, if $P_w(\omega=0)=1$ is provided. This regulator is integrating only if \mathcal{X}_d brings the pole $z=1$ into the denominator.

Because \mathcal{Y}_d can be considered as the numerator, and \mathcal{P} as the denominator of a closed-loop reference model P'_w

$$P'_w = \frac{\mathcal{Y}_d}{\mathcal{P}} \quad (11)$$

and investigating the sensitivity and complementary sensitivity functions of the *one-degree of freedom (ODOF)* closed-loop using the regulator (10), it is possible to form a *stabilizing two-degree of freedom (STDOF)* topology as shown in Fig. 3.

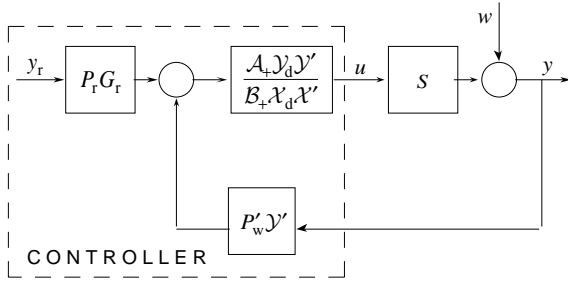


Fig. 3. The *STD OF* control system

It is easy to check that the tracking overall transfer function is

$$T_r = P_r G_r B_- z^{-d} \quad (12)$$

and the *sensitivity function (SF)* of the loop is

$$E = 1 - P'_w Y' B_- z^{-d} \quad (13)$$

So the closed-loop characteristics is

$$\begin{aligned} y_t &= T_r y_r + E w = y_t + y_d = \\ &= P_r G_r B_- z^{-d} y_r + (1 - P'_w Y' B_- z^{-d}) w \end{aligned} \quad (14)$$

In this topology the embedded filter G_r can be chosen arbitrary, so the same optimality can be achieved as it was for the case of open-loop stable plants. If we choose the embedded filter as $G'_w = Y'$, it comes from the *DE*, that the regulatory dynamics can not be optimized in the same way. This analysis means that the classical *DE* technique, even in the most general form, capable to solve the required optimality only for the tracking properties of a *TDOF* control system, but not for the disturbance rejection.

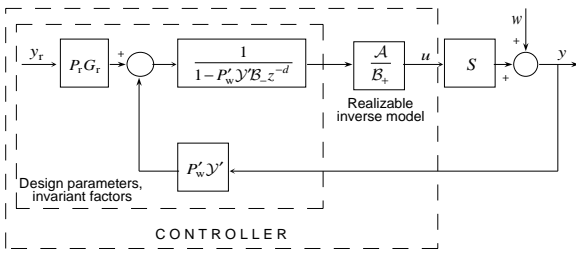


Fig. 4. Simplified equivalent form of the *STD OF* control system

Using further simplifications the regulator in (10) can also be expressed in another form

$$R = \frac{A_+ Y_d Y'}{B_+ X_d X'} = \frac{\left(\frac{Y_d}{P}\right) Y' A}{B_+ \left(1 - \frac{Y_d}{P} Y' B_- z^{-d}\right)} = \frac{P'_w G'_w S_+^{-1}}{1 - P'_w G'_w B_- z^{-d}} \quad (15)$$

which is formally exactly the same as (1), with the major difference that the embedded filter $G'_w = Y'$ is not selectable, it is given by the *DE* itself. This form of the *STD OF* control system is shown in Fig. 4. This equivalent form of the *STD OF* control system stands only for explanation and not for realization, because the indicated cancellation via A is not performed in the practice.

4. A Simple Error Decomposition

In a practical case only the model M of the process is known. Introduce the additive and relative model errors

$$\Delta = S - M \quad ; \quad \ell = \frac{\Delta}{M} = \frac{S - M}{M} \quad (16)$$

the *complementary sensitivity function (CSF)* of a *one-degree of freedom (ODOF)* control system is

$$T = \frac{RS}{1 + RS} = \hat{T} \frac{1 + \ell}{1 + \hat{T} \ell} \quad ; \quad \hat{T} = \frac{RM}{1 + RM} \quad (17)$$

where \hat{T} is the *CSF* of the model based *ODOF* system. The *SF* ($E = 1 - T$) can be decomposed into additive components according to different principles:

$$\begin{aligned} E &= (1 - P_w) + (P_w - \hat{T}) - (T - \hat{T}) = E_{\text{des}} + E_{\text{real}} + E_{\text{id}} = \\ &= (1 - P_w) + (P_w - T) = E_{\text{des}} + E_{\text{perf}} = (1 - \hat{T}) + (T - \hat{T}) = \\ &= E_{\text{cont}} + E_{\text{id}} = \hat{E} - (T - \hat{T}) = \hat{E} + E_{\text{id}} \end{aligned} \quad (18)$$

Here E_{des} is the design, E_{real} is the realizability, E_{id} is the identification (or modeling) degradations, respectively. Furthermore E_{perf} is the overall performance, E_{cont} is the overall control degradations, respectively. The *SF* depends on the model-based *SF* ($\hat{E} = 1 - \hat{T}$) as

$$E = \frac{1}{1 + RS} = \hat{E} \frac{1}{1 + \hat{T} \ell} \quad ; \quad \hat{E} = \frac{1}{1 + RM} \quad (19)$$

The term E_{id} can be further simplified

$$E_{\text{id}} = E - \hat{E} = \hat{T} - T = -\frac{\hat{T} \hat{E}}{1 + \hat{T} \ell} = -\hat{T} E \ell \Big|_{\ell \rightarrow 0} \approx -\hat{T} \hat{E} \ell \quad (20)$$

It is easy to see that $|\hat{T} \hat{E}|$ has its maximum at the cross over frequency ω_c , which means that the model minimizing E_{id} is the most accurate around this medium frequency range. (The realizability and identification degradations can be called as systematic (E_{sys}) and random (E_{rand}) components, too.)

For a *TDOF* system it is required to follow the transients prescribed by P_r and P_w (more exactly $(1-P_w)$), i.e. the ideal overall transfer function of the *GTDOF* control system would be

$$y^o = P_r y_r - (1-P_w) w = y_r^o + y_w^o \quad (21)$$

while the realizable control provides only

$$y = T_r y_r - E w = T_r y_r - (1-T) w \quad (22)$$

Express the deviation between the ideal y^o and the best reachable (realizable and optimal) closed-loop as

$$\begin{aligned} \Delta y &= y^o - y = (P_r - T_r) y_r - (P_w - T) w = \\ &= E_{\text{perf}}^r y_r - E_{\text{perf}}^w w \end{aligned} \quad (23)$$

where E_{perf}^r is the performance degradation for tracking and $E_{\text{perf}}^w = E_{\text{perf}}$ is the performance degradation for the disturbance rejection (or control) behaviors, respectively.

Assuming that the overall *CSF* of a *TDOF* control system is $T_r = FT$, then similar decomposition can be introduced for the tracking error function $E_r = 1 - T_r$ as for E in (18):

$$E_r = (1 - P_r) + (P_r - \hat{T}_r) - (T_r - \hat{T}_r) = E_{\text{des}}^r + E_{\text{real}}^r + E_{\text{id}}^r \quad (24)$$

The overall transfer function of the *TDOF* system is

$$T_r = \hat{T}_r \frac{1 + \ell}{1 + \hat{T} \ell} \quad (25)$$

The term E_{id}^r can be further simplified

$$E_{\text{id}}^r = \hat{T}_r - T_r = -\frac{\hat{T}_r \hat{E}}{1 + \hat{T} \ell} = -\hat{T}_r E \ell \Big|_{\ell \rightarrow 0} \approx -\hat{T}_r \hat{E} \ell \quad (26)$$

Youla-parametrization

If the applied regulator design is based on the *YP*, shown in section II, then the regulators in the true closed-loop and in the model based closed-loop are

$$R = \frac{Q}{1 - QS} \quad ; \quad \hat{R} = \frac{Q}{1 - QM} \quad (27)$$

Thus the *CSF* of the *ODOF* control system is

$$T = \frac{\hat{R}S}{1 + \hat{R}S} = \frac{QM(1 + \ell)}{1 + QM \ell} \quad ; \quad \hat{T} = \frac{RM}{1 + RM} = QM \quad (28)$$

In case of the *YP* one can also compute an ideal *CSF*, as

$$T_o = \frac{RS}{1 + RS} = QS = QM(1 + \ell) = \hat{T}(1 + \ell) \quad (29)$$

The *SF* of the true and model based closed-loops are

$$\hat{E} = \frac{1}{1 + \hat{R}M} = 1 - QM \quad (30)$$

and

$$E = \frac{1}{1 + \hat{R}S} = \frac{1 - QM}{1 + QM \ell} = \frac{\hat{E}}{1 + \hat{T} \ell} \quad (31)$$

The ideal *SF*, corresponding to T_o is

$$E_o = \frac{1}{1 + RS} = 1 - QS = 1 - QM(1 + \ell) = \hat{E} - \hat{T} \ell \quad (32)$$

The decomposition of the *SF* is now

$$\begin{aligned} E &= (1 - P_w) + (P_w - \hat{T}) - (T - \hat{T}) = E_{\text{des}} + E_{\text{real}} + E_{\text{id}} = \\ &= (1 - P_w) + (P_w - QM) - \frac{QM(1 - QM)}{1 + QM \ell} \ell \end{aligned} \quad (33)$$

where the identification degradation is

$$E_{\text{id}} = -\frac{QM(1 - QM)}{1 + QM \ell} \ell \Big|_{\ell \rightarrow 0} = -QM(1 - QM) \ell \quad (34)$$

It is interesting to note that for the ideal case the decomposition of $E_o = 1 - T_o$ results in

$$\begin{aligned} E_o &= E_{\text{des}} + E_{\text{real}} + E_{\text{id}}^o = \\ &= (1 - P_w) + (P_w - QM) - QM \ell = \\ &= E_{\text{des}} + E_{\text{perf}} = (1 - P_w) + (P_w - QS) \end{aligned} \quad (35)$$

where

$$E_{\text{id}}^o = -QM \ell \quad (36)$$

This last expression is different from the form (34), because at the optimal point, when $M = S$ the *Y-parametrized* closed-loop virtually opens, therefore the weighting by \hat{E} is missing.

The decomposition of the tracking error function for the *YP*

$$\begin{aligned} E_r &= 1 - T_r = (1 - P_r) + (P_r - Q_r M) - (T_r - \hat{T}_r) = \\ &= E_{\text{des}}^r + E_{\text{real}}^r + E_{\text{id}}^r \end{aligned} \quad (37)$$

where

$$E_{\text{id}}^r = -\frac{Q_r M(1 - QM)}{1 + QM \ell} \ell \Big|_{\ell \rightarrow 0} = -Q_r M(1 - QM) \ell \quad (38)$$

It is important to note that the term E_{real} can be made zero for IS processes, however, for IU plants the reachable minimal value of E_{real} always depends on the invariant factors and never becomes zero.

One possibility to optimize any norm formulated for the closed-loop SF is to consider a criterion as a function of the loop-parameters (design, regulator, constraints, etc.) and to solve the strongly nonlinear constrained mathematical programming problem. The existing advanced software tools are capable to solve such complex tasks, however give relatively little understanding for the influence of the different factors. This is why we prefer and propose such decomposition of the original problem, when the separate tasks are easy to be understood and well scaled in the selection parameters and factors.

The goal of the minimization of $\|E_{\text{des}}\|$, i.e. the design loss connected to the design degradation, is to find the fastest reference model $P_w = P_w^{\text{opt}}$ under the existing constraints for the control signal $u \in \mathcal{U}$. The goal of the minimization of $\|E_{\text{id}}\|$, i.e. the identification loss connected to the modeling degradation, is to find an optimal reference signal series y_r^{opt} (input design), minimizing a selected norm. These procedures usually result in certain minimax problems, where the optimized "maximum variance" type excitation produces the worst modeling error, to be minimized in the next identification step [5]. The goal of the minimization of $\|E_{\text{real}}\|$, i.e. the realizability loss connected to the realizability degradation, is discussed in the sequel. This decomposed task corresponds to the classical "model-matching" [8] type optimization of closed-loop systems. The $\|E_{\text{real}}\|$ loss depends only on the model of the process, on the reference model P_w (and on P_r for the dual problem for tracking), and the selected norm, so it is a "compact" optimization problem.

5. An Iterative Procedure to Optimize the Regulatory Properties of $STDOF$ Systems

Assume that the discrete-time model M , used in the pole-placement, is factorable as the process in (4)

$$M = M_+ \overline{M}_- = M_+ M_- z^{-d_m} = \frac{\hat{B}_+ \hat{B}_-}{\hat{A}} z^{-d_m} \quad (39)$$

where $M_+ = \hat{B}_+ / \hat{A}$ means the IS , $M_- = \hat{B}_-$ the IU factors, respectively. Assume that the denominator of the unstable model (see (7)) can be factorized as $\hat{A} = \hat{A}_+ \hat{A}_-$, where \hat{A}_+ contains the stable and \hat{A}_- does the unstable poles.

It is important to recall that the optimization of a serial embedded filter G_w , optimally attenuating the influence of the invariant factor M_- in \mathcal{H}_2 and \mathcal{H}_∞ norm spaces, requires a structure $G_w = 1/\mathcal{G}$ if $M_- = \hat{B}_-$ [4]. E.g., the polynomial $\mathcal{G} = \hat{B}_-^- = \hat{B}_-(z = z^{-1})$ is obtained for the \mathcal{H}_2 optimality, providing an all-pass filtering effect in the \hat{B}_- / \hat{B}_-^- compensation for the $GTDOF$ system.

In the regulatory part of the $STDOF$ closed-loop characteristics it is observable that the original invariant factor \hat{B}_- is augmented via the virtual factor $\mathcal{Y} \hat{B}_-$. Assuming that \mathcal{Y} is factorable into an IS part \mathcal{Y}_+ and an IU part \mathcal{Y}_-

$$\mathcal{Y} = \mathcal{Y}_+ \mathcal{Y}_- \quad (40)$$

then the proper selection of G'_w is

$$G'_w = \frac{1}{\mathcal{G}} = \frac{1}{\mathcal{G}_+ \mathcal{G}_-} = \frac{1}{\mathcal{Y}_+ \mathcal{G}_-} \quad (41)$$

where \mathcal{G}_- can be obtained from the \mathcal{H}_2 optimal attenuation of the virtual factor $\mathcal{Y} \hat{B}_-$.

If we want \mathcal{G} to appear in the equation (14) properly the DE (9) should be changed to

$$\begin{aligned} (\hat{A} \mathcal{X}_d \mathcal{G}) \mathcal{X}' + (\hat{B}_- z^{-d} \mathcal{Y}_d) \mathcal{Y}' &= \mathcal{G} \mathcal{P} \\ \hat{A}' \mathcal{X}' + \hat{B}' \mathcal{Y}' &= \mathcal{P}' \end{aligned} \quad (42)$$

The obtained regulator is slightly changed to

$$\hat{R} = \frac{\mathcal{Y}}{\mathcal{X}} = \frac{\hat{A}_+ \mathcal{Y}_d \mathcal{Y}'}{\mathcal{G} \hat{B}_+ \mathcal{X}_d \mathcal{X}'} \quad (43)$$

which causes the model-based SF to change as

$$\begin{aligned} \hat{E} &= 1 - P'_w G'_w \mathcal{Y} \hat{B}_- z^{-d} = 1 - P'_w \frac{\mathcal{Y}_+ \mathcal{Y}_- \hat{B}_-}{\mathcal{G}_+ \mathcal{G}_-} z^{-d} = \\ &= 1 - P'_w \frac{\mathcal{Y}_- \hat{B}_-}{\mathcal{G}_-} z^{-d} \end{aligned} \quad (44)$$

Now we can form an iterative procedure to find the optimal $STDOF$ regulator:

1. Solve the reduced DE (9) using the initial model factors \hat{A}_- and \hat{B}_- to obtain the initial \mathcal{X}' and \mathcal{Y}' .
2. Calculate $\mathcal{Y} \hat{B}_-$ and solve the optimization in \mathcal{H}_2 and/or \mathcal{H}_∞ norm spaces to get \mathcal{G}_- [4].
3. Compute $\mathcal{G} = \mathcal{Y}_+ \mathcal{G}_-$ and form the modified reduced DE (42).
4. Solve the reduced DE (9) to obtain \mathcal{X}' and \mathcal{Y}' .

5. The iterative process is continued from step 2, while a stop condition is not fulfilled (until the ultimate control objective is achieved or it is terminated by reaching some vital constraints).

6. Finding Stable Stabilizing Regulators

The *DE* based stabilizing regulator can result in unstable regulator. In this design approach it can not be guaranteed that the regulator is stable. Theoretically this is not a problem, because the stability of the closed-loop is the most important requirement. It is, however, sometimes required in practical applications that the stabilizing regulator should fall into the class of stable regulators. One of the advantage of the previously discussed methodology is that the required closed-loop transients are designed by two reference models and not by two weighting filters (usually applied in classical *LQ*-like problem formulations) or by two loop-shaping filters (usually applied in mixed \mathcal{H}_2 - \mathcal{H}_∞ problems). The practical applications of the algorithms, discussed in the previous sections, proved that it is possible to find stable stabilizing regulators if our design goal formulated by P_w requires less demanding closed-loop performance. It is definitely right, if the goal of the stabilization is to replace the unstable poles by their mirrored (over the unit circle) stable poles, and the other stable poles remain unchanged.

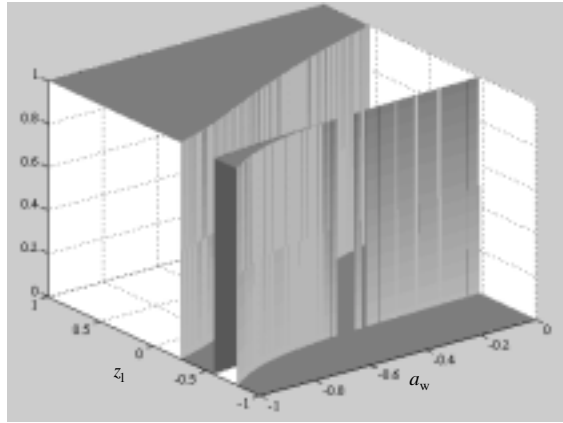


Fig. 5. Stable regulator domain for plant (45)

The stable regulator domain can not be obtained by explicit algebraic algorithms, however, it is possible to use advanced visualization tools based on specific numerical methods to localize these parameter areas.

It is not so difficult to write such MATLAB™ program, which scans the parameter range of P_w (practically the required bandwidth represented by the corner frequency of the reference model) to plot the acceptable stable regulator domain. Note that this "optimization" of P_w is different from the

optimization of $\|E_{des}\|$ and usually provides much slower closed-loop, than the fastest possible P_w .

Example 1.

Consider a simple discrete-time control system, where the unstable plant transfer function is

$$S = -\frac{0.2}{(1-z_1)} \frac{(1-z_1 z^{-1})}{(1-0.8z^{-1})(1-1.2z^{-1})} \quad (45)$$

The unity gain reference model is

$$P_w = \frac{(1+a_w)z^{-1}}{1+a_w z^{-1}} = \frac{(1+a_w)}{z+a_w} \quad (46)$$

where in our investigations the parameter domains of a_w and the zero z_1 cover the intervals

$$a_w = \{-1, \dots, 0\} \quad ; \quad z_1 = \{-1, \dots, 1\} \quad (47)$$

Fig. 5 indicates the obtained acceptable regulator domain, where the stabilizing regulator is stable (plotted value equal to one and zero corresponds to stabilizing, but unstable regulators). It can be well seen that for slower, less demanding reference model the parameter range of the zeros is much wider, where the regulator is stable.

Example 2.

Consider a faster unstable plant having transfer function

$$S = -\frac{0.8}{(1-z_1)} \frac{(1-z_1 z^{-1})}{(1-0.2z^{-1})(1-1.2z^{-1})} \quad (48)$$

and use the same reference model P_w as in (46). The stable stabilizing regulator (plotted value equal to one) can be reached for a wider range of the zero z_1 .

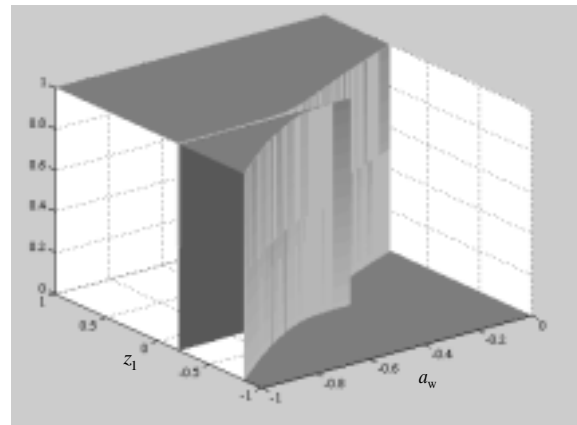


Fig. 6. Stable regulator domain for plant (48)

7. Conclusions

One of the most classical stabilizing methods for open-loop unstable plant is the state feedback observer topology. Unfortunately this method can not be applied for time-delay systems. Because the time-delay for discrete-time systems leaves the transfer functions of the plant in the class of rational functions, it is possible to use the *DE* solution based techniques for such processes.

It is investigated here how the optimality, besides the stabilizing pole-placement, can be ensured in a *TDOF* scheme. The computation possibilities are compared to *Youla-parametrization* based *GTDOF* systems introduced for open-loop stable plants. It is proved that for the tracking properties there is practically no difference between the two one-step approaches, besides the different regulator computation algorithms. However, for the disturbance rejection (control) problem the optimization of the regulator can be reached only by an iterative (multiple-step) method.

The paper analyses the internal structure of the sensitivity function and it is shown that the realizability degradation part can be optimized by these "model-based" class of optimization procedures.

It is also investigated and demonstrated by simulation how stable stabilizing regulators can be obtained by the proper selection of the selected reference model for design. The determination of these parameter domains needs to use advanced visualization tools based on appropriate numerical methods.

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