On Reliability Analysis of Cost-effective Hybrid Zeta Network : A Fault-tolerant Multi-stage Interconnection Network

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Abstract - *This paper presents the reliability studies of fault-tolerant Zeta Networks (ZTNs): a MIN, designed specially for an irregular class of hybrid networks. The reliability of ZTNs under Dynamic Full Access (DFA) is studied. The metrics used are mean time to failure (MTTF) and cost-effectiveness. Studies of tight Upper Bound (UB) and Lower Bound (LB) of the MTTF shows that ZTNs are more reliable and better in comparison to the ABNs, regular fault-tolerant multipath MIN and the QTNs, an irregular fault-tolerant multipath MIN. Cost-effectiveness studies shows that ZTNs are cost-effective as when compared with regular multipath fault-tolerant MIN.*

Keywords: Multi-stage interconnection networks, fault-tolerance, reliability, dynamic full access, dead-fault model, MTTF.

1 Introduction

Multi-stage Interconnection Networks (MINs) have been studied for broadband switching and for multiprocessor interconnection applications and in addition to this MINs always offer an attractive way of implementing fast packet switches in communication networks. With the throughput requirement of the packet switches exceeding several gigabits/sec, it becomes imperative to make them fault-tolerant. The topological design of MINs may be regular and irregular. Various researchers done sufficient work on regular type of fault-tolerant MINs, but irregular fault-tolerant MINs never were been in limelight. Any network is said to be irregular if the number of switching elements (SEs) in each stage of the network are different.

A number of techniques have been used to increase the reliability and fault-tolerance of MINs, a survey of the fault-tolerance attributes of these networks is found in [1]. The modest cost of unique paths MINs makes them attractive for large multiprocessors systems, but their lack of fault-tolerance, is a major drawback. To mitigate this problem, three hardware options are available: (1) Replicate the entire network, (2) Add extra stages and /or (3) Add chaining links.

Three, Zeta Networks [2], Augmented Baseline Networks (ABNs) [3] and Quad-tree Networks (QTNs) [4,5], Fault-tolerant multiple path MINs are used as running examples throughout the paper. A reliability analysis of these MINs is done to evaluate the MTTF as well as the Cost-effectiveness. The analytic bounds, cost, and cost-effectiveness are corroborated by simulation.

The rest of the paper is organized as follows: Section 2 describes the hardware topology of ABNs, QTNs and ZTNs. Fault-tolerance and reliability aspects are discussed in section 3. The cost and cost-effectiveness of ZTNs is analyzed in Section 4. Finally, the conclusions are given in Section 5.

2 Regular and Irregular Interconnection network

This section provides brief description of various types of network based on their regular and irregular topologies.

2.1 Augmented Baseline Network – a regular MIN

Any network is said to be regular if the number of switching elements (SEs) in each stage of the network are same.

2.1.1 Network definition of ABN

A 16 x 16 ABN (Figure (1)), consists of two identical subnetworks each consisting of N/2 sources (S) and equal number of destinations (D). Links are provided among switches belonging to the same stage, forming several loops of switches of size 3 x 3 in all stages except the last one where the SEs are of size 2 x 2, and they are connected to multiplexers (MUX) and demultiplexers (DEMUX) at the input stage and at the output stage respectively.

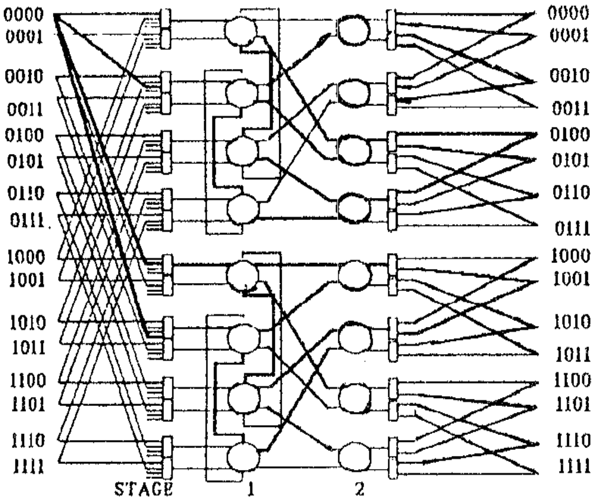

Figure 1. ABN of size 16 x 16

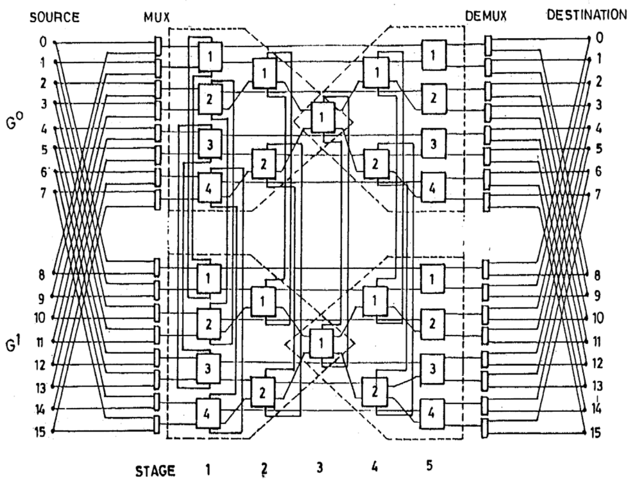

Figure 2. QTN of size 16 x 16

2.2 Quad-tree Network – an irregular MIN

The QTNs are having an irregular topology.

2.2.1 Network definition of QTN

A 16 x 16 QTN, (Figure (2)) is constructed by using two identical groups each consisting of modified double tree networks (MDOTs) [4,5] of size N/2 x N/2, which are arranged one above the other. Loops are formed by the switches having the same number in the same stage, which are formed in all the stages except the last one. All the SEs is of 3 x 3 in size except the ones in the last stage that are 2 x 2. Each source and destination is connected to both the groups through MUX and DEMUX. Alternate paths are available for the data from an input port to reach its final output port, which are selected based on an algorithm [5] that bypasses a faulty or a busy SE. The delay of the network is directly proportional to the path length encountered on the way to the destination.

2.3 Zeta Network – an irregular MIN

The QTNs and hybrid ZTNs are topological equivalent i.e. both are irregular in its design strategy. Any network is said to be hybrid if the number of SEs in different stages are of different sizes i.e. 2 x 2 and 3 x 3 etc.

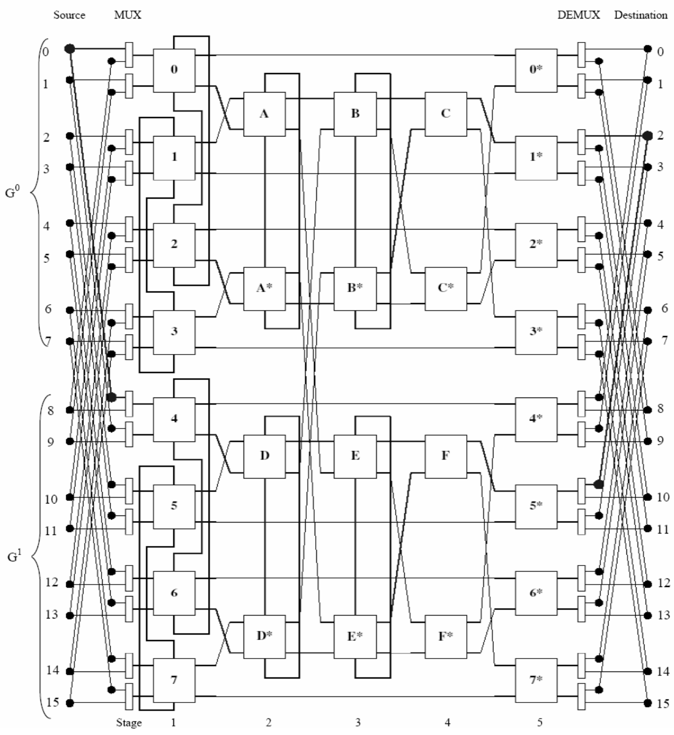

Figure 3. ZTN of size 16 x 16, highlighting the multiple paths between S-D pair

2.3.1 Network definition of ZTN

The idea of QTN is used in the designing of a 16 x 16 multipath ZTN, (Figure (3)), has extra SEs in intermediate stages with additional express chaining links, provide better fault-tolerance to the network. A ZTN of size 2^n x 2^n (where 2^n are Source, 2^n are Destination, $n = \log_2 N$, and $m = \log_2(N/2)$) is constructed with the help of two identical groups G^N, [where (N = 0, 1)], each consisting of a DOT network of size 2^{n-1} x 2^{n-1}, which are arranged one above the other. The two groups are formed based on the most significant bit (MSB) of the source-destination terminals. Thus, half of the source-destination terminals with MSB 0 falls into the G^0 group and the others having MSB 1 fall into G^1. Each source and destination is connected to both groups with the help of MUX and DEMUX.

3 Fault-tolerance and Reliability analysis of ZTNs–MTTF

There are three types of fault models adopted to measure the reliability of the MINs: 'the stuck-at fault model', 'the link-fault model', and 'the dead-fault model'. In the stuck-at fault model, a failure causes a crossbar switch to remain in a particular state regardless of the control inputs given to it, thus affecting its capability to setup suitable connections. In the link-fault model, a failure affects an individual link of a switch, leaving remaining part of the switch operational. In the dead-fault model, the strongest of the three, a failure of a switch makes it totally unusable and non-operational. In this paper, the dead-fault model is used for the analysis of the ZTNs. We assume that any of the switching components, i.e. crossbar switches, MUX or DEMUX, in ZTNs can fail. We also assume that faults are independent of each other.

A well-known criterion used to measure the reliability of fault-tolerant multi-stage interconnection networks is DFA, or the ability to route from any input to any output in a finite number of passes. Under this criterion, reliability of network can be measured in terms of MTTF (usually measured in terms of UB and LB. MTTF of the network is defined as the expected time elapsed before some sources are disconnected from some source [6]. ZTNs are fault-tolerant network, so before their analysis the number of faults that it can tolerated is worth being mentioned. A network is said to be k-fault tolerant, if it can still provide a connection for any source-destination pair in the presence of k faults in the network. Since ZTNs provide two disjoint paths for each source-destination pair, so ZTN are single switch fault-tolerant. However, one can find combination of two switches, which when simultaneously faulty, can disconnect a source from destination. For instance, if both the switches to which a source or a destination is connected becomes faulty, then that source or destination is disconnected from the rest of the network. However, if such critical combinations of switches are not present in a fault pattern, several multiple faults can be tolerated [2]. A ZTN of size 16, highlighting the multiple paths between S-D pair shown in Figure (3).

To make the analysis of MTTF tractable, we need to have some assumptions. We use the assumptions that have been made previously in other studies of fault-tolerant networks. The following assumptions are made during the analysis.

1. Not all the switching elements are dependent on each other. Therefore, it means failure of any switch does not affect the reliability of other.
2. The failure of switch occurs independently in a network with a failure rate of ($\lambda = 10^{-6}$ per hour) permit time.

Based on the gate counts of crossbar switches [7], the number of gates in a 2 x 2 crossbar switch is approximately equal to that in a 2 x 1 MUX or a 1 x 2 DEMUX. Thus to simplify the analysis we can assume that $\lambda_m = m\lambda/2$ for a m x 1 MUX, where λ_m failure rate of MUX or λ_d (= λ_m) for 1 x m DEMUX, where λ_d failure rate of DEMUX. The adaptive routing scheme of ZTN considers a 2 x 2 switch in the last stage and its associated DEMUX as a series system, so we consider these three elements as single component (SE_{2d}), and based on a gate count, a failure rate of $\lambda_{2d} = 2\lambda$ can be assigned to this group of elements. Also let λ_2 and λ_3 be the failure rate for the 2 x 2 (SE_2) and the 3 x 3 switch (SE_3), then based on gate count, $\lambda_2 = \lambda$ and $\lambda_3 = 2.25\lambda$ and $\lambda_{3m} = 4.25\lambda$.

3.1 Zeta Network

The bounds on reliability are computed by applying Series-Parallel model [6] of the concerned network. The MTTF for the Zeta network in terms of upper and lower bound is calculated in this section :

3.1.1 Upper Bound

The MTTF of ZTN can be analyzed by defining a critical set of components. A critical set of components is defined as the set of m+1 switching components, each from different groups, such that a network failure will occur if all the m+1 components become faulty simultaneously. It is observed that each source is connected to MUX in both the subnetworks. Thus, ZTN is operational as long as one of the two MUX attached to a source in either subnetwork is operational. This permits a simple reliability block diagram of the optimistic (upper) bound as shown in Figure (4(a)).

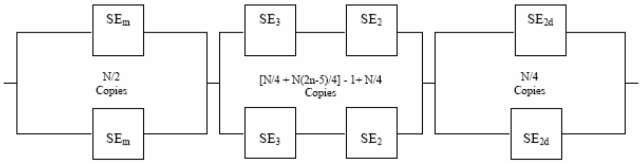

Figure 4(a). Reliability Block Diagram of ZTN for the Evaluation of MTTF in terms of Upper Bound

$$R_{ZTN-UB}(t) = [1-(1-e^{-\lambda_m t})^2]^{N/2} \\ [1-(1-e^{-\lambda_3 t})^2(1-e^{-\lambda_2 t})^2]^{(N/4+N(2n-5)/4)-1+N/4} \\ [1-(1-e^{-\lambda_{2d} t})^2]^{N/4} \quad (1)$$

$$^{*}MTTF_{ZTN-UB} = \int_0^\infty R_{ZTN-UB}(t)dt \quad (2)$$

3.1.2 Lower Bound

For lower bound, each group is considered independently and is assumed faulty if there is any single fault in it. Since at the input side of ZTN, routing scheme does not consider the MUX to be the integral part of the 2 x 2 switch. Hence, if both MUX are grouped with each switch in the input side and regarded, as a series system, then we will have the conservative estimate of the reliability of these components. To obtain the pessimistic (lower) bound of the reliability of ZTN we assume that the network is failed whenever more than one conjugate loop [2] has a faulty conjugate or more than one conjugate fails in the last stage. The reliability block diagram for lower bound MTTF is shown in Figure (4(b)).

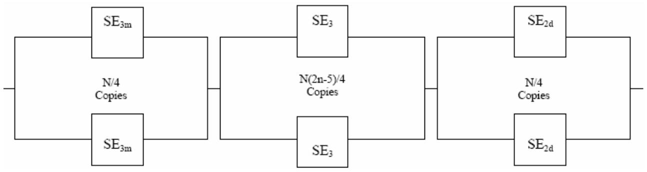

Figure 4(b). Reliability Block Diagram of ZTN for the Evaluation of MTTF in terms of Lower Bound

$$R_{ZTN-LB}(t) = [1-(1-e^{-\lambda_{3m} t})^2]^{N/4} \\ [1-(1-e^{-\lambda_3 t})^2]^{N(2n-5)/4}\,[1-(1-e^{-\lambda_{2d} t})^2]^{N/4} \quad (3)$$

$$^{*}MTTF_{ZTN-LB} = \int_0^\infty R_{ZTN-LB}(t)dt \quad (4)$$

3.2 Augmented Baseline Network

The MTTF for the Augmented Baseline network in terms of upper and lower bound is:

3.2.1 Upper Bound

$$R_{ABN-UB}(t) = [1-(1-e^{-\lambda_m t})^2]^{N/2} \\ [1-(1-e^{-\lambda_3 t})^2]^{N(n-3)/4}\,[1-(1-e^{-\lambda_{2d} t})^2]^{N/4} \quad (5)$$

$$^{*}MTTF_{ABN-UB} = \int_0^\infty R_{ABN-UB}(t)dt \quad (6)$$

3.2.2 Lower Bound

$$R_{ABN-LB}(t) = [1-(1-e^{-2\lambda_{3m} t})^2]^{N/8} \\ [1-(1-e^{-2\lambda_3 t})^2]^{N(n-4)/8}\,[1-(1-e^{-\lambda_{2d} t})^2]^{N/4} \quad (7)$$

$$^{*}MTTF_{ABN-LB} = \int_0^\infty R_{ABN-LB}(t)dt \quad (8)$$

3.3 Quad-tree Network

The MTTF for the Quad-tree network in terms of upper and lower bound is:

3.3.1 Upper Bound

$$R_{QTN-UB}(t) = [1-(1-e^{-\lambda_m t})^2]^{N/2} \\ [1-(1-e^{-\lambda_3 t})^2]^{N/4+N/4+N/8+.....+1} \\ [1-(1-e^{-\lambda_{2d} t})^2]^{N/4} \quad (9)$$

$$^{*}MTTF_{QTN-UB} = \int_0^\infty R_{QTN-UB}(t)dt \quad (10)$$

3.3.2 Lower Bound

$$R_{QTN-LB}(t) = [1-(1-e^{-\lambda_{3m} t})^2]^{N/4} \\ [1-(1-e^{-\lambda_3 t})^2]^{N/4+N/8+.....1(n-3)} \\ [1-(1-e^{-\lambda_{2d} t})^2]^{N/4} \quad (11)$$

$$^{*}MTTF_{QTN-LB} = \int_0^\infty R_{QTN-LB}(t)dt \quad (12)$$

All *MTTF Equation (2, 4, 6, 8, 10, and 12) are integrated in the interval t = 0.001 to 1 and are simulated with the help of MATLAB version 7.0.1. The values for upper and lower bound of MTTF for different regular and irregular multi-path MINs are provided in Table-1. The cost-functions and cost-effectiveness of the various networks are solved using the same software and values are provided in Table-2 and Table-3. The Figures [5-6] are resulted from the values (obtain during the simulation of respective MTTF in terms of upper and lower bound, cost functions and cost-effectiveness) provided in Table [1-3].

Table 1
Upper and Lower Bound values of MTTF for different sizes

Networks	MTTF vs. Size N			
	N=8	N=16	N=64	N=256
ABN-UB	329455.26	156006.78	51270.11	20335.21
ABN-LB	204035.83	96237.34	32050.54	12937.73
ZTN-UB	246859.17	147688.00	66069.34	28510.18
ZTN-LB	166463.24	117468.28	52778.10	24965.76
QTN-UB	162152.43	112824.68	54337.19	26462.33
QTN-LB	145533.16	93820.37	43570.81	21103.36

3.4 Comparisons - On Reliability

Relative variations in upper and lower bound MTTF of ABNs, ZTNs, and QTNs, are shown in Figure (5). From the Figure (5) and Table-1 it can be depicted that the difference in reliability of the three MINs indicates that for large network sizes (>N=8), upper bound and lower bound MTTF of ZTNs has an edge over upper and lower bounds MTTF of other fault-tolerant multi-path networks, implies that ZTNs are more reliable in comparison to ABNs and QTNs and can tolerate greater number of faults.

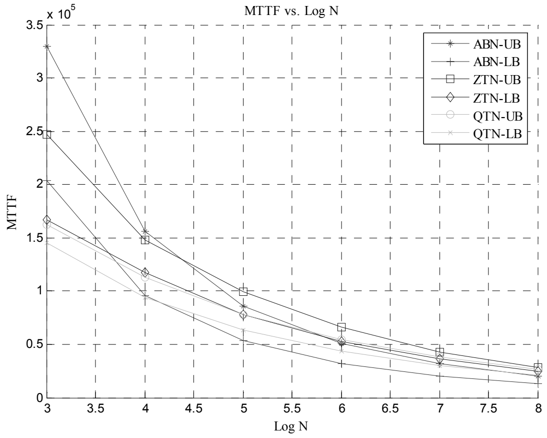

Figure 5. Comparative Upper and Lower Bounds of the MTTF of related MINs

4 Cost-effectiveness

To estimate the hardware cost of a network , one common method is to calculate the switch complexity with the assumption that the cost of a switch is proportional to the number of gates involved, which is roughly proportional to the number of ‘crosspoints’ within a switch [7]. For example, a 4 x 4 switch has 16 units of hardware cost whereas a 2 x 2 switch has 4 units. For the MUX and DEMUX, we roughly assume that each of K x 1 MUX or 1 x K DEMUX has K units of cost. In this way ABNs, QTNs, and ZTNs have cost of $(9n - 11)N/2$, $(9.75\ 2^{n+1} - 54)$, and $N/8(56+24\log_2(N/2))$ respectively. The values cost function for different regular and irregular multi-path MINs are provided in Table-2. From the Table-2, it is clear that ZTNs are more cost-effective than ABNs. This advantage becomes more significant as the network size increases.

Table 2
Cost values of different networks for different sizes

Networks	Cost vs. Size N				
	N=4	N=16	N=64	N=256	N=1024
ABN	14	200	1376	7808	40448
ZTN	40	256	1408	7168	34816
QTN	24	258	1194	4938	19914

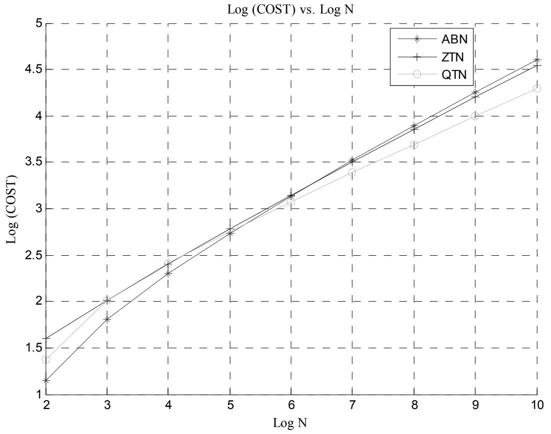

Figure 6(a). Comparative cost of related MINs

Now, a simple measure of the cost-effectiveness for reliability can be given by comparing MTTF and the cost of the network. Let the cost-effectiveness, η, of a network for reliability be the ratio of MTTF to its cost. To highlight the cost-effectiveness, the cost-effectiveness of the ABNs, QTNs, and ZTNs (for both upper and lower bounds) are evaluated and compared, and the improvement in results are shown in Figure (6(b)). From the results, we can observe that ZTNs are more cost-effective than most of the other fault-tolerant regular and irregular networks.

Table 3
Cost-effectiveness of different networks for different sizes

Networks	η = MTTF/Cost vs. Size N					
	N=8	N=16	N=32	N=64	N=128	N=256
ABN-UB	5147.73	780.03	158.34	37.26	9.57	2.60
ABN-LB	3188.06	481.18	98.19	23.29	6.03	1.65
ZTN-UB	2582.06	627.71	177.05	51.13	14.6	4.33
ZTN-LB	1600.60	458.86	128.40	37.48	11.34	3.48
QTN-UB	1589.73	437.30	137.26	45.50	15.50	5.35
QTN-LB	1426.79	363.64	111.12	36.49	12.38	4.27

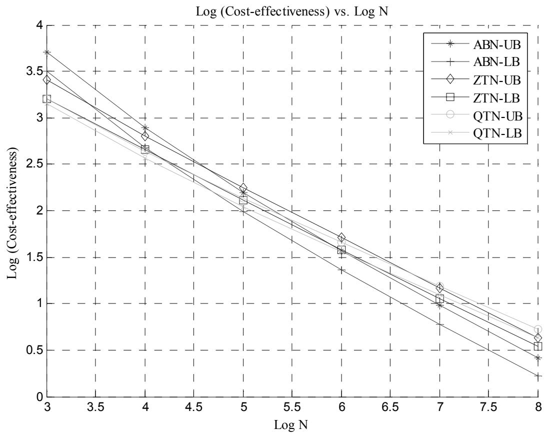

Figure 6(b). Comparative cost-effectiveness of related MINs

5 Conclusions

The reliability and cost-effectiveness analysis of the hybrid ZTNs with ABNs and QTNs shows a remarkable improvement in reliability, which not only illuminate the high-speed operation of the MIN but also compromising too much on hardware cost and density of the network. This multipath hybrid ZTN follow adaptive fault-tolerant routing provided by exploiting the inherent redundancy of the topology, which does not lead to blocking of request and hence mitigate the problem of faults in the network. The dead-fault model and DFA is used to study the reliability of the ZTNs. Figures of upper and lower bounds of MTTF shows that ZTN are more reliable and better in comparison to other fault-tolerant regular and irregular multipath MINs. cost-effectiveness shows that ZTNs are cost-effective as when compared with regular multipath fault-tolerant MINs.

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