Embedding Algorithm Between the Macro-star Graph and the Matrix-star Graph

***Kim jeong-suk** [1], Seo jung-hyun [2], Sim Hyun [3], Lee HyeongOk [4],

[1] Department of Computer Science, Oklahoma State University, USA

rockhee@hanmail.net

[2,3] Department of Computer Science, Sunchon National University, 315 Maegok-dong, Sunchon, Chonnam, 540-742, Korea

{jhseo@sunchon.ac.kr, simhyun@sunchon.ac.kr}, 82-61-752-3625

[4] Department of Computer Education, Sunchon National University, 315 Maegok-dong, Sunchon, Chonnam, 540-742, Korea

oklee@sunchon.ac.kr, 82-61-750-3345

Index Term – Interconnection Network, Macro-star, Matrix-star, Embedding Algorithm, Star Graph

Abstract

The Macro-star graph which has the star graph as a basic module has node symmetry, maximum fault tolerance, and hierarchical decomposition property. And, it is an interconnection network which improves a network cost against the star graph. A Matrix-star graph also has such a good properties of the Macro-star graph and is an interconnection network which has a lower network cost than the Macro-star graph.

In this paper, we propose a method to embed between the Macro-star graph and a Matrix-star graph. We show that the Macro-star graph MS(k,n) can be embedded into a Matrix-star graph $MS_{k,n+1}$ with dilation 2. In addition, we show that a Matrix-star graph $MS_{k,n}$ can be embedded into the Macro-star graph MS(k,n+1) with dilation 4 and average dilation 3 or less as well. This result means that several algorithms developed in the star graph can be simulated in a Matrix-star graph at a lower cost.

1. Introduction

Parallel processing computer is classified multiprocessor which has shared-memory and multi-computer which uses distributed-memory. In the multi-computer system, each processor has its own local memory and is connecting by an interconnection network.

* *This work was supported by the Korea Research Foundation Grant funded by the Korean Government (MOEHRD, Basic Research Promotion Fund) (R05-2004-000-11669-0)*

The communication between the processor is processed by the message transaction method through interconnection network. The multi-computer interconnection networks affected to the performance and extensibility of total system[5].

Such interconnection network is usually modeled as an undirected graph where the set of nodes represents the processors and the set of edges represents the bidirectional communication links between the processors. The network estimation to test the interconnection networks is network costs[11,12], which multiply degree by diameter. Interconnection networks are classified by the number of nodes, such as meshes[5,10] with $k \times n$ nodes, hypercubes[10,11] with 2^n nodes, star graphs[2,3] with $n!$ nodes.

The star graph has lower degree and smaller diameter than the hypercube, but it has limitations which the embedding with other networks is difficult and with the increase of node by increasing the dimension is very large. Therefore SCC[8], (n,k)-star graph[3], Bubblesort Star graph[4], Transposition graph[9], HCN[1] etc. have been proposed. When it has same nodes, Macro-star graph[11] and Matrix-star graph [12] which has lower degree and more improved network costs than star graph were also proposed. Macro-star graph having same basic module with the star graph has lower degree than the star graph, when it has same number of nodes. But it still has star graph's properties such as node symmetry, maximum fault tolerance, and hierarchical decomposition. Matrix-star graph has the good properties of Macro-star graph and is a better interconnection network which the network costs were improved.

The embedding is that a specific graph is mapping into other graph in order to know the relation of association and inclusion between the two graphs. What network G is possible to embed into network H at low cost is that all algorithms for network G can be efficiently used to network H at low cost. Therefore it is important to estimate an embedding of interconnection networks[5,6,10]. The $f=(\varphi,\rho)$, embedding graph G into graph H, is defined as following. The φ is a function which corresponds G's vertex set $V(G)$ to H's vertex set $V(H)$. The ρ is a function which corresponds G's edge $e=(v,w)$ to H path linking $\varphi(v)$ and $\varphi(w)$. The criterion to estimate the embedding costs is dilation, congestion, expansion and so on. The dilation of edge e of graph G means the length of path $\rho(e)$ on the graph H. The dilation of embedding f is the maximum value among all edge G. The congestion is the number of $\rho(e)$ including H's edge e'. The congestion of f means the maximum value among all edge's congestion of H. The expansion of embedding f means the ratio of G's vertex to H's vertex.

This paper analize the embedding between Macro-star graph and Matrix-star graph which the network cost is better improved than star graph. It also shows that various algorithm for the star graph can be efficiently simulated under the Matrix-star graph by analyzing the embedding relation between two graphs. Section 2 defines and discusses some of the basic properties of Macro-star graph and Matrix-star graph with dilation 2 and that Matrix-star can be embedded into Macro-star graph with dilation 4. Finally conclusions and future studies are presented.

2. Related work

2.1 The definition and properties of the Macro-star graph

Macro-star graph $MS(l,n)$ is the network with $(nl+1)!$ nodes and $(nl+1)!\times(n+l-1)!$ edges. The address of each nodes is expressed by the permutation of $k(=nl+1)$ different symbols. In the k bit string of node v and w, the edge exists between the permutations that is created by applying two edge generator T_j and S_i to the node w. When the k different symbol set is $\langle k\rangle=\{1,2,...,k\}$ and the symbol's permutation to the $\langle k\rangle$ is $U=u_{1:k}=u_1u_2...u_k$, $u_i\in\langle k\rangle$, the Macro-star graph is defined as following.

$$V(MS(l,n))=\{U=u_{1:k} \mid u_i, u_j \in\langle k\rangle, u_i\neq u_j, i\neq j, 1\leq i,j\leq k\}$$
$$E(MS(l,n))=\{(U,V) \mid U,V \in V(MS(l,n)) \text{ satisfying } U=S_i(V), 2\leq j\leq n+1, 2\leq i\leq l\}$$

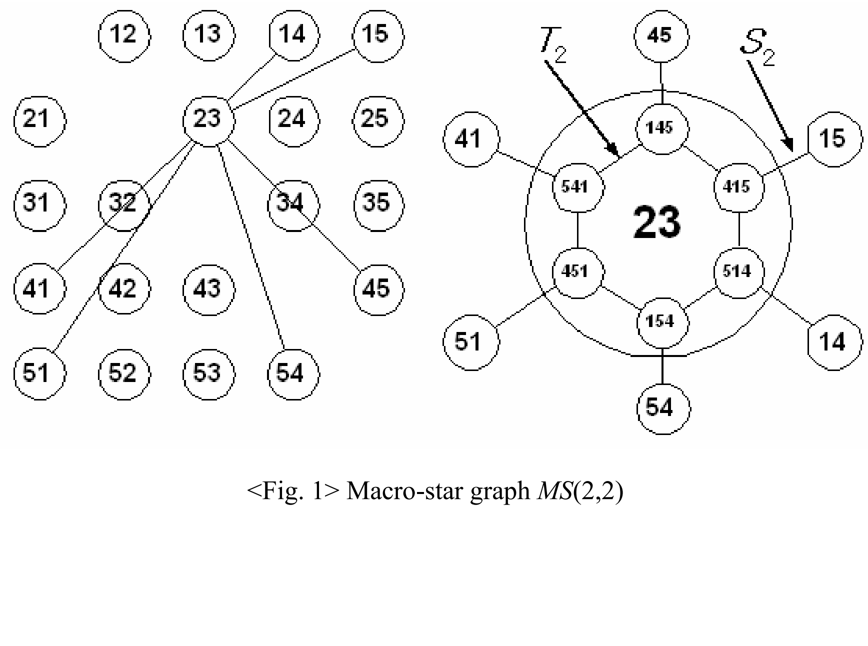
The edge generator T_j is an edge to connect permutation that is created by exchanging one element u_j among the first symbol u_1 and the first in the permutation $U(2\leq j\leq n+1)$.

$$T_j(U)=u_ju_{2:j-1}u_1u_{j+1}...u_k,\ j=2,3,...,n+1.$$

The edge generator $S_{n,i}$ is an edge to connect permutation that is created by exchanging the symbol sequence $u_{2:n+1}$ and $u_{(i-1)n+2:in+1}$ composing the first cluster and ith cluster in the permutation U.

$$S_{n,i}(U_{1:k})=u_1u_{(i-1)n+2:in+1}u_{n+2:(i-1)n+1}u_{2:n+1}u_{in+2:k},\ i=2,3,...,l.$$

The symbol sequence $u_{(i-1)n+2:in+1}$ means cluster(or level) and the number of element composing one cluster is n in the $MS(l,n)$. The edge generator is simply called S_i. <Fig.1> shows the node consisting Macro-star graph $MS(2,2)$ of 2th cluster.



<Fig. 1> Macro-star graph $MS(2,2)$

In the <Fig.1> the big circle on the right side is the node consisting 2th cluster's symbol of 23. The permutation of node 145 is 14523, the permutation created by the edge generator T_2(14523) in the permutation 14523 is 41523, and the permutation created by the edge generator T_3(14523) is 54123. If sequentially applying one permutation P to edge generator T_j and Si of Macro-star graph, it is represented to $S_iT_j(P)$, ($2 \le j \le n+1$, $2 \le i \le l$). In this paper, because the node of Macro-star graph represents the permutation to the $k(ln+1)$ symbols, the node and permutation have the same meaning.

2.2 The definition and properties of Matrix-star graph

The Matrix-star graph $MS_{k,n}$ represent the node, which is using the symbol 1,2,3, ..., $k \times n$, that is kth column nth row matrix following form's node.

$$\begin{bmatrix} x_1 & x_2 & \cdots & x_j & \cdots & x_n \\ x_{n+1} & x_{n+2} & \cdots & x_{n+j} & \cdots & x_{2n} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ x_{(i-1)n+1} & x_{(i-1)n+2} & \cdots & x_{(i-1)n+j} & \cdots & x_{in} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ x_{(k-1)n+1} & x_{(k-1)n+2} & \cdots & x_{(k-1)n+j} & \cdots & x_{kn} \end{bmatrix}$$

And the edge connecting nodes has the connecting relation between nodes represented by the following matrix($1 \le k \le n$).

(1) Matrix exchanged 1st column row for 1st column jth row.

$$\begin{bmatrix} x_j & x_2 & \cdots & x_1 & \cdots & x_n \\ x_{n+1} & x_{n+2} & \cdots & x_{n+j} & \cdots & x_{2n} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ x_{(i-1)n+1} & x_{(i-1)n+2} & \cdots & x_{(i-1)n+j} & \cdots & x_{in} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ x_{(k-1)n+1} & x_{(k-1)n+2} & \cdots & x_{(k-1)n+j} & \cdots & x_{kn} \end{bmatrix}$$

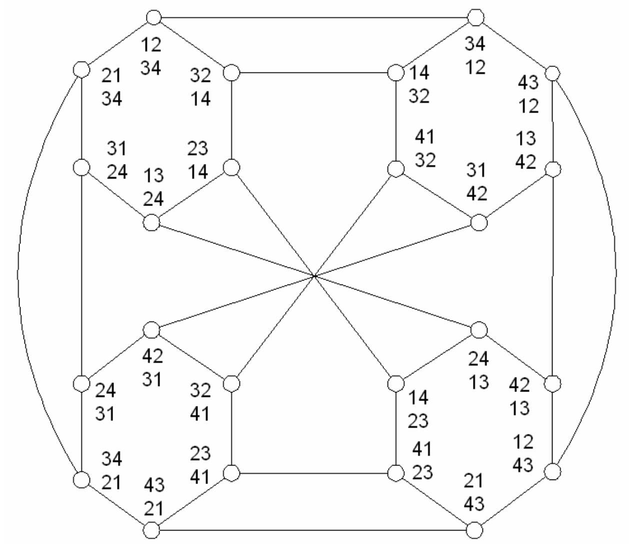
(2) Matrix exchanged 1st column row for 1st column ith row.

$$\begin{bmatrix} x_{(i-1)n+1} & x_2 & \cdots & x_j & \cdots & x_n \\ x_{n+1} & x_{n+2} & \cdots & x_{n+j} & \cdots & x_{2n} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ x_1 & x_{(i-1)n+2} & \cdots & x_{(i-1)n+j} & \cdots & x_{in} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ x_{(k-1)n+1} & x_{(k-1)n+2} & \cdots & x_{(k-1)n+j} & \cdots & x_{kn} \end{bmatrix}$$

(3) Matrix exchanged between the same column symbol for 1st row and ith row.

$$\begin{bmatrix} x_{(i-1)n+1} & x_{(i-1)n+2} & \cdots & x_{(i-1)n+j} & \cdots & x_{in} \\ x_{n+1} & x_{n+2} & \cdots & x_{n+j} & \cdots & x_{2n} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ x_1 & x_2 & \cdots & x_j & \cdots & x_n \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ x_{(k-1)n+1} & x_{(k-1)n+2} & \cdots & x_{(k-1)n+j} & \cdots & x_{kn} \end{bmatrix}$$

Let's divide the edge of Matrix-star graph $MS_{k,n}$ into three categories : the edge that is defined by the condition (1) is $C_j(2 \le j \le n)$, the edge that is defined by the condition (2) is $R_i(2 \le i \le k)$, the edge that is defined by the condition (3) is $E_i(2 \le i \le k)$. By the above definition, Matrix-star graph $MS_{k,n}$ can generate matrix as much as the number of permutation. Therefore it consists of $(k \times n)!$ nodes and the degree of node is $2k+n-3$. The $MS_{1,2}$ under $k=1$, $n=2$ is k_2 graph that is consisted by one degree and two nodes. The node of Matrix-star graph $MS_{k,n}$ is made up matrix form k columns $m \times n$ rows. The element of node is $k \times n$ and the element of i column j row in the $k \times n$ matrix of node s denotes s_{ij}. Fig.2 is an example of Matrix-star graph $MS_{2,2}$ that node denotes matrix of 2 columns 2 rows. Among the graphs suggested by hybrid star graph, Table.1 compares the Macro-star graph and Matrix-star graph that the network cost is most improved.



<Fig. 2> Macro-star graph $MS_{2,2}$

<Table 1> Compare the network cost

	Star S_{n^2}	Macro-star $MS(n,n)$	Matrix-star $MS_{k,k,k}$, $(k=\sqrt[3]{n^2})$
# node	$(n^2)!$	$(n^2+1)!$	$(n^2)!$
# degree	n^2-1	$2n-1$	$5(\sqrt[3]{n^2} - 1)$
# diameter	$\lfloor 3/2(n^2-1) \rfloor$	$2.5(n^2+n-1)$	$5n^2+4(\sqrt[3]{n^4})$
# network cost	$O(n^4)$	$O(n^3)$	$O(n^{2.7})$

3. Embedding the Macro-star *MS*(*k*,*n*) into a Matrix-star $MS_{k,n+1}$

The outline of this embedding is as follows. The matrix form, 1 column kn+1 row, that represents the node of Macro-star graph $MS(k,n)$ changes matrix form, k column n+1 row, that represents the node of a Matrix-star graph $MS_{k,n+1}$. The Macro-star $MS(k,n)$ graph consists of k clusters and each cluster consists of n symbols. Because the matrix representing the node of a Matrix-star graph $MS_{k,n+1}$ denotes k columns n+1 rows, the number of symbol of one column representing the node of a Matrix-star graph $MS_{k,n+1}$ is one more than the number of symbol of one cluster representing the node of Macro-star graph $MS(k,n)$. The symbols consisting the first cluster of Macro-star graph $MS(k,n)$ is located in the first column of the matrix $k(n+1)$ representing node expression form of Matrix-star graph and the symbol consisting ith cluster is located in the ith column($2 \le i \le k$). Because the number of symbol of one cluster of Macro-star graph $MS(k,n)$ is different from the number of symbol of one column of a Matrix-star graph, jth element of each cluster of the Macro-star graph $MS(k,n)$ makes to be located in the j+1th row of matrix $k(n+1)$ that represents as the node expression form of the Matrix-star graph($1 \le j \le n$). In the ith column of matrix $k(n+1)$ that represents as the node expression form of Matrix-star graph, Δ_i is assigned in the first row that symbol of Macro-star graph $MS(k,n)$ is not assigned($2 \le i \le k$). The following is a way of mapping the node of Matrix-star graph into the node of Matrix-star graph. First map the node U of Macro-star graph $MS(k,n)$ into node V of Matrix-star graph $MS_{k,n+1}$. And the mapping between symbols maps $u_{(i-1)n+a}$ into $v_{(i-1)n+j}$ in the remaining symbol except in the matrix $k(n+1)$ that represents the node of Macro-star graph, ($2 \le a \le n+1$, $i+1 \le j \le l+n$).

Theorem 1. The Macro-star graph $MS(k,n)$ can be embedded in the Matrix-star graph $MS_{k,n+1}$ with dilation 2 and expansion $(kn+1)!/(kn+k)!$.

proof When the permutation of node U of the Macro-star graph $MS(k,n)$ is $u_1 u_{2:n+1} u_{n+2:2n+1} \ldots u_{(i-1)n+2:in+1} \ldots u_{(k-1)n+2:kn+1}$, suppose the matrix that node U changes a node expression form $k(n+1)$ of Matrix-star graph $MS_{k,n+1}$ is node U'. The matrix $k(n+1)$ of node U' is as follows :

$$\begin{bmatrix} v_1 & v_2 & \cdots & v_j & \cdots & v_n & v_{n+1} \\ \Delta_2 & v_{n+2} & \cdots & v_{n+j} & \cdots & v_{2n} & v_{2n+1} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \\ \Delta_i & v_{(i-1)n+2} & \cdots & v_{(i-1)n+j} & \cdots & v_{in} & v_{in+1} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \\ \Delta_l & v_{(k-1)n+2} & \cdots & v_{(k-1)n+j} & \cdots & v_{kn} & v_{kn+1} \end{bmatrix}$$

The node U of Macro-star graph $MS(k,n)$ is connecting to the node $S_i(U)$ and the node $T_j(U)$ by the edge generator S_i and T_j($2 \le i \le k$, $2 \le j \le n+1$). The embedding will be analyzed by the fact that the node $S_i(U)$ and the node $T_j(U)$ of the Macro-star graph has the same permutation with the permutation generated by applying the edge of the Matrix-star graph $MS_{k,n+1}$ to the node U' that changes node U of Macro-star graph $MS(k,n)$ into the node expression form of Matrix-star graph $MS_{k,n+1}$.

The adjacent node to the node U of Macro-star graph $MS(k,n)$ is connecting by the edge generator S_i and T_j. Therefore two kinds of edge generator is introduced.

Case1 Edge generator $T_{j+1}(1 \le j \le n)$

The edge generator T_{j+1} of Macro-star graph $MS(k,n)$ denotes the edge C_j of the Matrix-star graph $MS_{k,n+1}(1 \le j \le n)$. In the permutation of the node U of Macro-star graph $MS(k,n)$ $u_1 u_{2:n+1} u_{n+2:2n+1} \ldots u_{(i-1)n+2:in+1} \ldots u_{(k-1)n+2:kn+1}$, the node $T_{j+1}(U)$ connecting by the edge generator T_{j+1} is the permutation $u_1 u_{2:n+1} u_{n+2:2n+1} \ldots u_{(i-1)n+2:in+1} \ldots u_{(k-1)n+2:kn+1}$ mutually exchanging the first symbol u_1 of node U and jth symbol of the first cluster. The node $T_{j+1}(U')$ that changes node $T_{j+1}(U)$ of Macro-star graph $MS(k,n)$ into the node expression form of Matrix-star graph $MS_{k,n+1}$ is as following.

$$\begin{bmatrix} v_j & v_2 & \cdots & v_1 & \cdots & v_n & v_{n+1} \\ \Delta_2 & v_{n+2} & \cdots & v_{n+j} & \cdots & v_{2n} & v_{2n+1} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \\ \Delta_i & v_{(i-1)n+2} & \cdots & v_{(i-1)n+j} & \cdots & v_{in} & v_{in+1} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \\ \Delta_l & v_{(k-1)n+2} & \cdots & v_{(k-1)n+j} & \cdots & v_{kn} & v_{kn+1} \end{bmatrix}$$

In the node U, because the position of symbols that can be exchanged by the edge generator T_{j+1} is the symbol from the second position to $(n+1)$th position, the first symbol of first column and the $(j+1)$th symbol is same with the exchanged permutation. It can be mapping the node U of Macro-star graph $MS(k,n)$ into the node V of Matrix-star graph $MS_{k,n+1}$ and mapping the node $T_{j+1}(U)$ adjacent to the node U into the node $C_{j+1}(U')$. Therefore the edge generator T_{j+1} of the Macro-star graph $MS(k,n)$ serves as the edge C_{j+1} of the Matrix-star graph $MS_{k,n+1}$.

Case 2 Edge generator $S_i(2 \le i \le l)$

The edge generator S_i of Macro-star graph $MS(k,n)$ denotes the edge E_i and R_i of Matrix-star graph $MS_{k,n+1}(2 \le i \le k)$. In the permutation $u_1 u_{2:n+1} u_{n+2:2n+1} \ldots u_{(i-1)n+2:in+1} \ldots u_{(l-1)n+2:ln+1}$ of the node U of the Macro-star graph $MS(k,n)$, the node $S_i(U)$ connecting by the edge generator S_i is the permutation $u_1 u_{(i-1)n+2:in+1} \ldots u_{2:n+1} \ldots u_{(l-1)n+2:ln+1}$ that consists by exchanging the first cluster's n symbols $u_{2:n+1}(=u_2 u_3 \ldots u_{n+1})$ and ith cluster's n symbols $u_{(i-1)n+2:in+1}$. Being changed the permutation $S_i(U)$ connecting by the edge generator S_i to matrix form same as the node form of the Matrix-star graph $k(n+1)$, the node $S_i(U')$ is as follows.

$$\begin{bmatrix} u_1 & u_{(i-1)n+2} & \cdots & u_{(i-1)n+j} & \cdots & u_{in} & v_{in+1} \\ \Delta_2 & u_{n+2} & \cdots & u_{n+j} & \cdots & v_{2n} & v_{2n+1} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \\ \Delta_i & u_2 & \cdots & u_j & \cdots & u_n & u_{n+1} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \\ \Delta_l & u_{(l-1)n+2} & \cdots & u_{(l-1)n+j} & \cdots & u_{ln} & u_{ln+1} \end{bmatrix}$$

Permutation $S_i(U)$, which is connected by node U, can be constructed the same permutation $R_i(E_i(U'))$, which is generated to node U' by star graph edge E_i and R_i. Therefore permutation $S_i(U)$ have the same permutation $R_i(E_i(U'))$. We can show that the transform of permutation node U' by the edge S_i and R_i of Matrix-star graph.
First, we can apply the edge E_i of Matrix-star graph to node U', and the result permutation node $E_i(U')$ is as follows.

$$\begin{bmatrix} \triangle_i & u_{(i-1)n+2} & \cdots & u_{(i-1)n+j} & \cdots & u_{in} & v_{in+1} \\ \triangle_2 & u_{n+2} & \cdots & u_{n+j} & \cdots & v_{2n} & v_{2n+1} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \\ u_1 & u_2 & \cdots & u_j & \cdots & u_n & u_{n+1} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \\ \triangle_l & u_{(l-1)n+2} & \cdots & u_{(l-1)n+j} & \cdots & u_{ln} & u_{ln+1} \end{bmatrix}$$

Second, the permutation node $R_i(E_i(U'))$ is generated by the edge R_i of Matrix-star graph to node U'. The following matrix is the node $R_i(E_i(U'))$.

$$\begin{bmatrix} u_1 & u_{(i-1)n+2} & \cdots & u_{(i-1)n+j} & \cdots & u_{in} & v_{in+1} \\ \triangle_2 & u_{n+2} & \cdots & u_{n+j} & \cdots & v_{2n} & v_{2n+1} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \\ \triangle_i & u_2 & \cdots & u_j & \cdots & u_n & u_{n+1} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \\ \triangle_l & u_{(l-1)n+2} & \cdots & u_{(l-1)n+j} & \cdots & u_{ln} & u_{ln+1} \end{bmatrix}$$

In the permutation $u_1u_{2:n+1}u_{n+2:2n+1} \ldots u_{(i-1)n+2:in+1} \ldots u_{(l-1)n+2:ln+1}$ of the node U of Macro-star graph $MS(k,n)$, the node $S_i(U)$ connecting by the edge generator S_i is the permutation $u_1u_{(i-1)n+2:in+1} \ldots u_{2:n+1} \ldots u_{(l-1)n+2:ln+1}$ that consists by exchanging the first cluster's n symbols $u_{2:n+1}(=u_2u_3 \ldots u_{n+1})$ and ith cluster's n symbols $u_{(i-1)n+2:in+1}$.

4. Conclusion

In this paper, it was analyzed the embedding between Macro-star graph and Matrix-star graph having more improved network cost with important properties of interconnecting network: node symmetry, recursive structure, maximum fault tolerance than star graph. This paper shows that the Macro-star graph $MS(k,n)$ can be embedded into the Matrix-star graph $MS_{k,n+1}$ with dilation 2 and expansion $(kn+1)!/(kn+k)!$ and that matrix-star graph $MS(k,n)$ can be embedded into the Macro-star graph $MS(k,n+1)$ with dilation 4 and expansion $(kn)!/(kn+k+1)!$. At the above result, the average dilation rate of embedding Matrix-star graph $MS(k,n)$ into Macro-star graph $MS(k,n+1)$ was less than 3.

The result of embedding Macro-star graph into Matrix-star graph with dilation 2 means that various algorithm developed for star graph can be simulated with the extra constant cost under the Matrix-star graph, because Macro-star graph has star graph as a basic module.

Reference

[0] S.K. Yun and K.H. Park, "Hierarchical Cubic Networks", IEEE Trans. Parallel Distributed syst., Vol. 9, No. 4, pp.410-414, 1998.

[1] S. B. Akers and B. Krishnamurthy, "On Group Graphs and Their Fault Tolerance," IEEE Trans. Comput., Vol. c-36, No. 7, pp. 885-888, July 1987.

[2] N. Corp, "NCUBE/ten : an Overview," November 1985.

[3] D. R. Duh, G. H. Chen and J. F. Fang, "Algorithms and Properties of a New Two-Level Network with Folded Hypercubes as Basic Modeles," IEEE Trans. Parallel Distributed syst., Vol. 6, No. 7, pp.714-723, 1995.

[4] A. El-Amawy and S. Latifi, "Properties and Performance of Folded Hypercubes," IEEE Trans. Parallel Distributed syst., Vol. 2, No. 1, pp.31-42, 1991.

[5] K. Ghose and K. R. desai " Hierarchical Cubic Networks," IEEE Trans. Parallel Distributed syst., Vol.6, No. 4, pp.427-436, 1995.

[6] F. Harary, J. P. Hayes, and H.-J. Wu, "A Survey of the Theory of Hypercube Graphs," Comput. Math. Appl., Vol. 15, pp.277-289, 1988

[7] F. T. Leighton, Introduction to Parallel Algorithms and Architectures : Arrays, Hypercubes, Morgan Kaufmann Publishers, 1992.

[8] V. E. Mendia nad D. Sarkar, "Optimal Broadcasting on the Star Graph," IEEE Trans. Parallel Distributed syst., Vol.3, No.4, pp.389-396, 1992.

[9] S. Ranka, Y. Won and S. Sahni, "Programming a Hypercube Multiprocessor," IEEE Software, Vol. 5, pp. 69-77, 1988.

[10] Y. Saad and M.H. Schutltz, "Topological Properties of Hypercubes," IEEE Trans. Comput., Vol.37, pp.867-872, 1988.

[11] A. S. Vaidya, P. S. N. Rao and S. R. Shankar, "A Class of Hypercube_like Networks," Proc. of the 5th IEEE Symposium on Parallel and Distributed Processing, pp. 800-803, Dec. 1993.

[12] H.-O. Lee, J.-S. Kim, K.-W. Park, J.-S. Seo, and E. Oh, "Matrix-star Graphs: A New Interconnection Network Based on Matrix Operation," 10th Asia-Pasific Conference, LNCS 3740, pp. 478-487, 2005.