Hamiltonian Paths and Cycles in Faulty Burnt Pancake Graphs

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Abstract ***Recently, research on parallel processing systems is very active, and many complex topologies have been proposed. A burnt pancake graph is one such topology. In this paper, we prove that a faulty burnt pancake graph with degree*** n ***has a fault-free Hamiltonian cycle if the number of faulty elements is no more than*** $n-2$***, and it has a fault-free Hamiltonian path between any pair of non-faulty nodes if the number of faulty elements is no more than*** $n-3$***.***

Keywords: burnt pancake graph, interconnection networks, Hamiltonian path, Hamiltonian cycle, parallel processing

1 Introduction

Ongoing researches on parallel and distributed computation and on massively parallel processing have led to proposals for interconnection networks based on complicated topologies, such as a hypercube, a star graph, a pancake graph, and a rotator graph. Most of those new topologies are based on the Cayley graph and have been the focus of many intensive research efforts[1, 2, 4, 5, 7, 10, 14]. A burnt pancake graph[8] is a such new topology. It can provide the different number of nodes from others based on the signed permutations, which can be used to the genome analysis[9].

A massively parallel system has many elements such as nodes and links (edges). Hence, it is necessary to design algorithms on the assumption of the existence of faulty elements. Therefore, in this paper, we consider the case that a burnt pancake graph with degree n has faulty elements and prove that it has a fault-free Hamiltonian cycle if the number of the faulty elements is at most $n-2$, and that it has a fault-free Hamiltonian path between any pair of nonfaulty nodes if the number of the faulty elements is at most $n-3$[3, 6, 11, 12, 13, 15, 16].

For an n-pancake graph, previous work has proved that it has a fault-free Hamiltonian cycle if the number of the faulty elements is at most $n-3$, and that it has a Hamiltonian path between any pair of nonfaulty nodes if the number of the faulty elements is at most $n-4$[13].

The rest of this paper is structured as follows. Section 2 gives several definitions. In Section 3, we present two lemmas with their proofs, and In Section 4, we prove the main theorem. Section 5 describes the conclusions and the future work.

2 Preliminaries

In this section, we introduce the definitions of signed permutations, prefix reversal operations, burnt pancake graphs, and Hamiltonian connectivity.

Definition 1 (Signed Permutation) $\boldsymbol{u} = (u_1, u_2, \cdots, u_n)$ where $\{|u_1|, |u_2|, \cdots, |u_n|\} = \{1, 2, \cdots, n\}$ is called a signed permutation of n integers $1, 2, \cdots, n$.

Definition 2 (Prefix Reversal Operation) Let $\boldsymbol{u} = (u_1, u_2, \cdots, u_n)$ be a signed permutation of n integers $1, 2, \cdots, n$. Then, the operations $\boldsymbol{u}^{(i)}$ $(1 \leq i \leq n)$ defined by $\boldsymbol{u}^{(i)} = (-u_i, -u_{i-1}, \cdots, -u_1, u_{i+1}, \cdots, u_n)$ are called prefix reversal operations.

We denote the successive applications of the prefix reversal operations $(\boldsymbol{u}^{(i_1)})^{(i_2)}$ by $\boldsymbol{u}^{(i_1,i_2)}$ and $(\boldsymbol{u}^{(i_1,i_2,\cdots,i_{k-1})})^{(i_k)}$ by $\boldsymbol{u}^{(i_1,i_2,\cdots,i_{k-1},i_k)}$. In the rest of this paper, the negative sign is placed at the top of the expressions, say $\overline{u}_1$, to save space.

Definition 3 (Burnt Pancake Graph with Degree n, BP_n) A burnt pancake graph with degree n, BP_n, is an undirected graph, which has $n! \times 2^n$ nodes. Each node has a unique label $\boldsymbol{u} = (u_1, u_2, \cdots, u_n)$ that is a signed permutation of n integers $1, 2, \cdots, n$. For two nodes $\boldsymbol{u}$ and $\boldsymbol{v}$, there exists an edge between them if and only if there exists i $(1 \leq i \leq n)$ such that $\boldsymbol{u}^{(i)} = \boldsymbol{v}$.

Figure 1 shows BP_3. A BP_n contains $2n$ disjoint subgraphs BP_{n-1}. All nodes in each of these subgraphs have a common integer as the final elements of their labels. Therefore, each subgraph can be specified $BP_{n-1}k$ by using the common integer k. Note that if an edge $(\boldsymbol{u}, \boldsymbol{v})$ between two distinct subgraphs exists, it is always given by the operation $\boldsymbol{v} = \boldsymbol{u}^{(n)}$.

Table 1 gives comparison of BP_n with a pancake graph P_n, a rotator graph R_n, a star graph S_n, a hypercube Q_n, a de Bruijn graph $B_{n,k}$, and a Kautz graph $K_{n,k}$. If we define the performance measurement of interconnection networks by (No. of Nodes)/((Degree) $\times$ (Diameter)), BP_n is superior to P_n, R_n, S_n, $T_{n,k}$, $M_{n,k}$ and Q_n. Though $B_{n,k}$ and $K_{n,k}$ superior to BP_n, they do not have symmetric nor recursive structures that are necessary to execute several parallel and distributed algorithms.

Definition 4 For an arbitrary pair of nodes in a graph G, if there is a Hamiltonian path between them, G is called Hamiltonian connected.

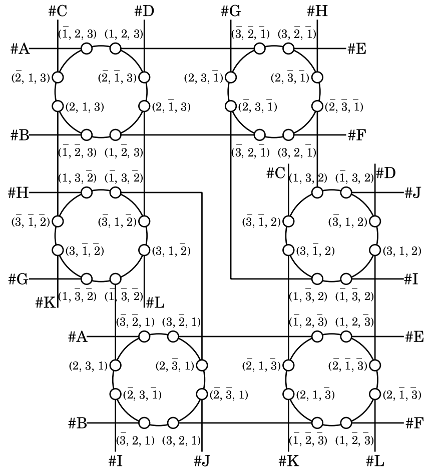

Figure 1: An example of a burnt pancake graph, BP_3.

Table 1: Comparison of a burnt pancake graph with other graphs.

	#Nodes	Degree	Connect.	Diameter
BP_n	$n! \times 2^n$	n	n	$\leq 2n+3$
P_n	$n!$	$n-1$	$n-1$	†
R_n	$n!$	$n-1$	$n-1$	$n-1$
S_n	$n!$	$n-1$	$n-1$	$\lfloor \frac{3(n-1)}{2} \rfloor$
Q_n	2^n	n	n	n
$B_{n,k}$	n^k	n	n	k
$K_{n,k}$	$n^k + n^{k-1}$	n	n	k

†: $\leq \lceil 5(n+1)/3 \rceil$ from [8].

3 Lemmas

In this section, we introduce two lemmas that are necessary to prove the main theorem.

Lemma 1 Assume that F is the set of the faulty elements, that is, the faulty nodes and the faulty edges, in BP_n and $|F| \leq n-2$. Then, if each of m subgraphs $BP_{n-1}k_1 - F$, $BP_{n-1}k_2 - F$, $\cdots$, $BP_{n-1}k_m - F$ $(n \geq 4, m \geq 5)$ is Hamilton-connected, then the subgraph induced by these subgraphs is also Hamilton-connected.

(Proof) Assume that $\boldsymbol{u} \in BP_{n-1}k_1 - F$ and $\boldsymbol{v} \in BP_{n-1}k_m - F$. Then, from $m \geq 5$, we can rearrange $k_1, k_2, \cdots, k_m$ to obtain $k'_1, k'_2, \cdots, k'_m$ such that $k'_1 = k_1$, $k'_m = k_m$, $k'_i \neq \overline{k}'_{i+1}$ $(1 \leq i \leq m-1)$. Let $\boldsymbol{s}_1 = \boldsymbol{u}$ and $\boldsymbol{d}_m = \boldsymbol{v}$. For

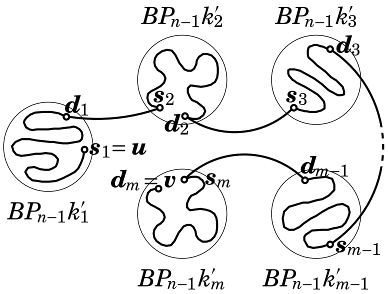

Figure 2: Construction of a Hamiltonian path for a set of subgraphs.

$i = 1, 2, \cdots, m-2$, execute the following two steps in this order: (1) first, select node $\boldsymbol{d}_i$ in $BP_{n-1}k'_i - F$ such that $\boldsymbol{d}_i^{(n)} \in BP_{n-1}k'_{i+1} - F$, and select Hamiltonian path $HP_{k'_i}(\boldsymbol{s}_i, \boldsymbol{d}_i)$ and edge $(\boldsymbol{d}_i, \boldsymbol{d}_i^{(n)})$; (2) next, let $\boldsymbol{s}_{i+1} = \boldsymbol{d}_i^{(n)}$. On the other hand, in $BP_{n-1}k'_m$, select node $\boldsymbol{s}_m$ that is not $\boldsymbol{d}_m$ and satisfies $\boldsymbol{s}_m^{(n)} \in BP_{n-1}k'_{m-1} - F$, and select Hamiltonian path $HP_{k'_m}(\boldsymbol{s}_m, \boldsymbol{d}_m)$ and edge $(\boldsymbol{s}_m^{(n)}, \boldsymbol{s}_m)$. Let $\boldsymbol{d}_{m-1} = \boldsymbol{s}_m^{(n)}$. Finally, in $BP_{n-1}k'_{m-1} - F$, select Hamiltonian path $HP_{k'_{m-1}}(\boldsymbol{s}_{m-1}, \boldsymbol{d}_{m-1})$. Now, Hamiltonian path $HP(\boldsymbol{u}, \boldsymbol{v})$ can be constructed between $\boldsymbol{u}$ and $\boldsymbol{v}$ as shown in Figure 2. □

Reversely, in BP_n, let $HP^{\{h_1, h_2, \cdots, h_m\}}(\boldsymbol{u}, \boldsymbol{v})$ represent the Hamiltonian path assured by Lemma 1 between two nodes $\boldsymbol{u}$ and $\boldsymbol{v}$ in the subgraph induced by the nodes other than m subgraphs $BP_{n_1}h_1$, $BP_{n-1}h_2$, $\cdots$, $BP_{n-1}h_m$.

Lemma 2 In faulty BP_3, assume that F is the set of faulty elements. Then, if $|F| \leq 1$, $BP_3 - F$ contains a Hamiltonian cycle. Additionally, if $|F| = 0$, $BP_3 - F$ is Hamiltonian connected.
(Proof) For the proof of the former part of this lemma, it is sufficient to show the existence of a Hamiltonian path for each case that F is node $\{(1,2,3)\}$, edge $\{((1,2,3),(\overline{1},2,3))\}$, edge $\{((1,2,3),(\overline{2},\overline{1},3))\}$, or edge $\{((1,2,3),(\overline{3},\overline{2},\overline{1}))\}$ by taking advantage of the symmetric property of BP_3. These Hamiltonian paths are in Table 2. In addition, for the proof of latter part, it is sufficient to

Table 2: Hamiltonian cycles in BP_3 with $|F| = 1$.

F	Hamiltonian Cycle
Node $(1,2,3)$	$(\overline{1},2,3), (\overline{2},1,3), (\overline{3},\overline{1},2), (3,\overline{1},2), (1,\overline{3},2), (\overline{1},\overline{3},2),$ $(3,1,2), (\overline{2},\overline{1},\overline{3}), (1,2,\overline{3}), (\overline{1},2,\overline{3}), (\overline{2},1,\overline{3}), (2,1,\overline{3}),$ $(\overline{1},\overline{2},\overline{3}), (1,\overline{2},\overline{3}), (2,\overline{1},\overline{3}), (3,1,\overline{2}), (\overline{3},1,\overline{2}), (\overline{1},3,\overline{2}),$ $(1,3,\overline{2}), (2,\overline{3},\overline{1}), (3,\overline{2},\overline{1}), (\overline{3},\overline{2},\overline{1}), (2,3,\overline{1}), (\overline{2},3,\overline{1}),$ $(\overline{3},2,\overline{1}), (3,2,\overline{1}), (\overline{2},\overline{3},\overline{1}), (1,3,2), (\overline{1},3,2), (\overline{3},1,2),$ $(\overline{2},\overline{1},3), (2,\overline{1},3), (1,\overline{2},3), (\overline{1},\overline{2},3), (2,1,3), (\overline{3},\overline{1},\overline{1}),$ $(3,\overline{1},\overline{2}), (1,\overline{3},\overline{2}), (\overline{1},\overline{3},\overline{2}), (2,3,1), (\overline{2},3,1), (\overline{3},2,1),$ $(3,2,1), (\overline{2},\overline{3},1), (2,\overline{3},1), (3,\overline{2},1), (\overline{3},\overline{2},1)$
Edge $((1,2,3),$ $(\overline{1},2,3))$	$(1,2,3), (\overline{3},\overline{2},\overline{1}), (2,3,\overline{1}), (1,\overline{3},\overline{2}), (\overline{1},\overline{3},\overline{2}), (3,1,\overline{2}),$ $(\overline{3},1,\overline{2}), (\overline{1},3,\overline{2}), (1,3,\overline{2}), (\overline{3},\overline{1},\overline{2}), (3,\overline{1},\overline{2}), (2,1,\overline{3}),$ $(\overline{1},\overline{2},\overline{3}), (1,\overline{2},\overline{3}), (2,\overline{1},\overline{3}), (\overline{2},\overline{1},\overline{3}), (1,2,\overline{3}), (3,\overline{2},\overline{1}),$ $(2,\overline{3},\overline{1}), (\overline{2},\overline{3},\overline{1}), (3,2,\overline{1}), (\overline{3},2,\overline{1}), (\overline{2},3,\overline{1}), (1,\overline{3},2),$ $(\overline{1},\overline{3},2), (3,1,2), (\overline{3},1,2), (\overline{1},3,2), (1,3,2), (\overline{3},\overline{1},2),$ $(3,\overline{1},2), (\overline{2},1,\overline{3}), (\overline{1},2,\overline{3}), (3,\overline{2},1), (2,\overline{3},1), (\overline{2},\overline{3},1),$ $(3,2,1), (\overline{3},2,1), (\overline{2},3,1), (2,3,1), (\overline{3},\overline{2},1), (\overline{1},2,3),$ $(\overline{2},1,3), (2,1,3), (\overline{1},\overline{2},3), (1,\overline{2},3), (2,\overline{1},3), (\overline{2},\overline{1},3)$
Edge $((1,2,3),$ $(\overline{2},\overline{1},3))$	$(1,2,3), (\overline{3},\overline{2},\overline{1}), (3,\overline{2},\overline{1}), (2,\overline{3},\overline{1}), (\overline{2},\overline{3},\overline{1}), (3,2,\overline{1}),$ $(\overline{3},2,\overline{1}), (\overline{2},3,\overline{1}), (2,3,\overline{1}), (1,\overline{3},\overline{2}), (\overline{1},\overline{3},\overline{2}), (3,1,\overline{2}),$ $(\overline{3},1,\overline{2}), (\overline{1},3,\overline{2}), (1,3,\overline{2}), (\overline{3},\overline{1},\overline{2}), (3,\overline{1},\overline{2}), (2,1,\overline{3}),$ $(\overline{2},1,\overline{3}), (\overline{1},2,\overline{3}), (1,2,\overline{3}), (\overline{2},\overline{1},\overline{3}), (2,\overline{1},\overline{3}), (1,\overline{2},\overline{3}),$ $(\overline{1},\overline{2},\overline{3}), (3,2,1), (\overline{3},2,1), (\overline{2},3,1), (2,3,1), (\overline{3},\overline{2},1),$ $(3,\overline{2},1), (2,\overline{3},1), (\overline{2},\overline{3},1), (\overline{1},3,2), (1,3,2), (\overline{3},\overline{1},2),$ $(3,\overline{1},2), (1,\overline{3},2), (\overline{1},\overline{3},2), (3,1,2), (\overline{3},1,2), (\overline{2},\overline{1},3),$ $(2,\overline{1},3), (1,\overline{2},3), (\overline{1},\overline{2},3), (2,1,3), (\overline{2},1,3), (\overline{1},2,3)$
Edge $((1,2,3),$ $(\overline{3},\overline{2},\overline{1}))$	$(1,2,3), (\overline{2},\overline{1},3), (2,\overline{1},3), (1,\overline{2},3), (\overline{1},\overline{2},3), (2,1,3),$ $(\overline{2},1,3), (\overline{3},\overline{1},2), (3,\overline{1},2), (1,\overline{3},2), (\overline{1},\overline{3},2), (3,1,2),$ $(\overline{3},1,2), (\overline{1},3,2), (1,3,2), (\overline{2},\overline{3},\overline{1}), (2,\overline{3},\overline{1}), (3,\overline{2},\overline{1}),$ $(\overline{3},\overline{2},\overline{1}), (2,3,\overline{1}), (\overline{2},3,\overline{1}), (\overline{3},2,\overline{1}), (3,2,\overline{1}), (1,\overline{2},\overline{3}),$ $(\overline{1},\overline{2},\overline{3}), (2,1,\overline{3}), (\overline{2},1,\overline{3}), (\overline{1},2,\overline{3}), (1,2,\overline{3}), (\overline{2},\overline{1},\overline{3}),$ $(2,1,\overline{3}), (3,1,\overline{2}), (\overline{3},1,\overline{2}), (\overline{1},3,\overline{2}), (1,3,\overline{2}), (\overline{3},\overline{1},\overline{2}),$ $(3,\overline{1},\overline{2}), (1,\overline{3},\overline{2}), (\overline{1},\overline{3},\overline{2}), (2,3,1), (\overline{2},3,1), (\overline{3},2,1),$ $(3,2,1), (\overline{2},\overline{3},1), (2,\overline{3},1), (3,\overline{2},1), (\overline{3},\overline{2},1), (\overline{1},2,3)$

show Hamiltonian path from node $(1,2,3)$ to all the other nodes from the symmetric property of BP_3. Examples of Hamiltonian paths are shown in Table 3. Note that this table gives the sequences of superscripts i of the prefix reversal operations $\boldsymbol{u}^{(i)}$ that generates a Hamiltonian path. □

4 Theorem

Theorem 1 In BP_n, let F be the set of faulty elements. Then, if $|F| \leq n-2$, $BP_n - F$ has a Hamiltonian cycle, and if $|F| \leq n-3$, $BP_n - F$ is Hamiltonian connected.
(Proof) If $n = 3$, the theorem holds from Lemma 2. Hence, we prove the theorem by mathematical induction assuming $n \geq 4$. First, we show the construction method for a Hamiltonian cycle in case that $|F| = n-2$.
Case A-1 $F \subset BP_{n-1}k_1$.

Table 3: Example of Hamiltonian Paths from $(1, 2, 3)$.

Dest.	Index Sequence of Prefix Reversal Operations to Generate Hamiltonian Path
$12\overline{3}$	1, 2, 1, 2, 1, 2, 1, 3, 2, 1, 2, 1, 2, 1, 2, 3, 1, 2, 1, 2, 1, 2, 3, 1, 2, 1, 2, 1, 2, 1, 3, 2, 1, 2, 1, 2, 1, 2, 3, 1, 2, 1, 2, 1, 2, 1, 3
$1\overline{2}3$	1, 2, 1, 2, 3, 1, 2, 1, 2, 1, 2, 1, 3, 2, 3, 2, 1, 2, 1, 2, 1, 2, 3, 2, 1, 2, 1, 2, 1, 2, 3, 1, 3, 2, 1, 2, 1, 2, 3, 1, 2, 1, 2, 1, 2, 1, 3
$1\overline{23}$	1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 1, 3, 2, 1, 2, 1, 2, 1, 2, 3, 1, 2, 1, 2, 1, 2, 1, 3, 2, 3, 1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 1, 3, 2, 1, 2, 1, 2
$\overline{1}23$	2, 1, 2, 1, 2, 1, 3, 1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 1, 2, 1, 2, 1, 3
$\overline{1}2\overline{3}$	1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 1, 2, 1, 2, 1, 3, 2, 1, 2, 1, 2, 1, 2, 3, 2, 1, 2, 1, 2, 1, 2, 3, 1, 2, 1, 2, 1, 2, 1, 3, 2, 1, 2, 1, 2, 1, 2
$\overline{12}3$	1, 2, 1, 3, 1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 1, 3, 2, 1, 3, 2, 1, 2, 1, 2, 3, 1, 2, 1, 3, 1, 2, 1, 2, 1, 2, 1, 3, 2, 3, 1, 2, 1, 2, 1, 2, 1, 3
$\overline{123}$	1, 2, 1, 2, 1, 2, 1, 3, 2, 1, 2, 1, 2, 3, 2, 1, 2, 1, 2, 1, 2, 3, 1, 3, 2, 1, 2, 1, 2, 1, 2, 3, 2, 3, 2, 1, 2, 1, 3, 1, 2, 1, 2, 1, 3, 2, 1
132	1, 2, 1, 2, 1, 2, 1, 3, 2, 3, 1, 3, 2, 1, 2, 1, 3, 2, 1, 3, 1, 3, 1, 2, 1, 2, 1, 3, 1, 2, 1, 3, 2, 1, 3, 2, 1, 2, 1, 3, 2, 1, 3, 1, 3, 1, 2
$13\overline{2}$	1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 1, 3, 2, 1, 2, 1, 2, 1, 2, 3, 1, 2, 1, 2, 3, 2, 1, 2, 1, 2, 1, 2, 3, 1, 2, 1, 3, 2, 1, 2, 1, 2, 1, 2, 3, 1, 2
$1\overline{3}2$	1, 2, 1, 2, 1, 2, 1, 3, 2, 1, 2, 1, 3, 1, 2, 1, 2, 1, 3, 2, 3, 2, 1, 2, 1, 3, 2, 1, 2, 3, 2, 1, 3, 1, 3, 1, 2, 3, 2, 1, 2, 3, 1, 2, 1, 2, 3
$1\overline{32}$	1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 3, 2, 1, 3, 1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 1, 2, 1, 2, 3, 2, 1, 2, 1, 3, 1, 2, 1, 2, 3, 2, 1, 2, 1, 2, 1, 2, 3
$\overline{1}32$	1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 1, 3, 2, 1, 2, 1, 3, 2, 1, 3, 2, 1, 2, 1, 2, 3, 1, 2, 3, 1, 2, 1, 2, 3, 1, 2, 1, 2, 1, 2, 1, 3, 2, 3, 1, 2, 1
$\overline{1}3\overline{2}$	1, 2, 1, 2, 1, 2, 1, 3, 2, 1, 2, 1, 2, 3, 2, 1, 2, 1, 3, 2, 1, 2, 3, 2, 1, 3, 1, 2, 1, 2, 1, 2, 1, 3, 2, 3, 2, 1, 2, 1, 2, 1, 2, 3, 2, 1, 2
$\overline{13}2$	1, 2, 1, 2, 1, 2, 1, 3, 1, 3, 1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 1, 2, 1, 2, 1, 3, 2, 1, 2, 1, 2, 1, 2, 3, 2, 1, 2, 1, 2, 1, 2, 3, 1, 2, 1, 2, 1
$\overline{132}$	1, 2, 1, 2, 1, 2, 1, 3, 2, 1, 2, 1, 2, 3, 2, 1, 2, 1, 2, 1, 2, 3, 2, 1, 2, 1, 2, 1, 3, 2, 1, 2, 1, 2, 1, 2, 3, 2, 3, 2, 1, 2, 1, 2, 1, 2, 3
213	1, 2, 3, 1, 2, 1, 2, 1, 3, 1, 2, 1, 3, 2, 1, 2, 1, 2, 3, 2, 1, 2, 1, 2, 3, 1, 2, 1, 2, 1, 2, 1, 3, 2, 3, 1, 3, 2, 1, 2, 1, 2, 1, 2, 3, 2, 3
$21\overline{3}$	1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 1, 2, 1, 2, 1, 3, 1, 3, 2, 1, 2, 1, 2, 1, 2, 3, 1, 2, 1, 2, 1, 2, 1, 3, 2, 1, 2, 1, 3, 1, 2, 1, 2, 1, 3, 2, 1
$2\overline{1}3$	1, 2, 1, 2, 1, 3, 1, 2, 1, 3, 2, 1, 2, 1, 2, 1, 2, 3, 1, 2, 1, 2, 1, 2, 1, 3, 1, 3, 1, 2, 1, 3, 1, 2, 3, 1, 2, 1, 2, 1, 3, 1, 2, 1, 2, 3, 1
$2\overline{13}$	1, 2, 1, 2, 1, 2, 1, 3, 2, 1, 2, 1, 2, 1, 2, 3, 2, 3, 1, 2, 1, 2, 1, 2, 1, 3, 2, 1, 2, 1, 2, 1, 2, 3, 1, 2, 1, 2, 1, 2, 1, 3, 2, 1, 2, 1, 2
$\overline{2}13$	1, 3, 1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 1, 3, 2, 1, 2, 1, 3, 2, 1, 3, 2, 1, 2, 1, 2, 1, 2, 3, 1, 2, 1, 3, 2, 1, 2, 1, 2, 1, 2, 3, 2, 1, 2, 1, 3
$\overline{2}1\overline{3}$	1, 2, 1, 2, 1, 2, 1, 3, 2, 1, 2, 1, 2, 3, 2, 1, 2, 1, 3, 2, 1, 2, 3, 2, 1, 3, 1, 3, 1, 2, 3, 2, 1, 2, 3, 1, 2, 1, 2, 3, 2, 3, 1, 2, 1, 2, 1
$\overline{21}3$	1, 2, 1, 2, 1, 2, 3, 1, 3, 2, 1, 2, 3, 1, 2, 1, 3, 2, 1, 2, 3, 1, 3, 2, 1, 2, 1, 2, 1, 2, 3, 1, 3, 2, 1, 2, 3, 1, 2, 1, 3, 2, 1, 2, 3, 1, 3
$\overline{213}$	1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 1, 2, 1, 2, 1, 3, 2, 1, 2, 1, 2, 1, 2, 3, 2, 1, 2, 1, 2, 1, 2, 3, 1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 1, 2, 1, 2, 1
231	1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 3, 2, 1, 3, 1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 1, 2, 1, 2, 1, 3, 2, 1, 2, 1, 2, 1, 2, 3, 2, 1, 2, 1, 3, 1, 2, 1, 2
$23\overline{1}$	1, 2, 1, 2, 1, 2, 1, 3, 2, 1, 2, 1, 2, 3, 2, 1, 2, 1, 3, 2, 1, 2, 1, 3, 2, 1, 3, 2, 1, 2, 1, 2, 3, 1, 2, 3, 1, 2, 1, 2, 3, 2, 3, 2, 3, 1, 2
$2\overline{3}1$	1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 1, 2, 1, 2, 1, 3, 2, 1, 2, 1, 2, 1, 3, 2, 1, 2, 1, 2, 1, 2, 3, 1, 2, 1, 2, 1, 2, 1, 3, 2, 1, 2, 1, 2, 1, 2, 3
$2\overline{31}$	1, 2, 1, 2, 1, 2, 1, 3, 2, 3, 1, 2, 1, 2, 3, 1, 3, 2, 1, 3, 1, 2, 1, 3, 1, 2, 1, 2, 1, 3, 1, 3, 2, 3, 2, 1, 3, 1, 3, 1, 2, 3, 2, 1, 2, 3, 1
$\overline{2}31$	1, 2, 1, 2, 1, 2, 1, 3, 2, 1, 2, 1, 2, 1, 2, 3, 2, 3, 1, 2, 1, 2, 1, 2, 1, 3, 2, 1, 2, 1, 3, 2, 1, 3, 2, 1, 2, 1, 2, 3, 1, 2, 3, 1, 2, 1, 2
$\overline{2}3\overline{1}$	1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 1, 2, 1, 2, 1, 3, 2, 1, 2, 1, 2, 1, 2
$\overline{23}1$	1, 2, 1, 2, 1, 2, 1, 3, 2, 1, 2, 1, 2, 1, 2, 3, 1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 1, 2, 1, 2, 1, 3, 2, 1, 2, 1, 2, 1, 2, 3, 2, 1, 2, 1, 2, 1, 2
$\overline{231}$	1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 1, 3, 2, 1, 3, 2, 1, 2, 1, 2, 3, 1, 2, 1, 3, 1, 2, 1, 2, 1, 2, 1, 3, 2, 3, 1, 2, 1, 2, 1, 2, 1, 3, 2, 1, 2, 1
312	1, 2, 1, 2, 1, 2, 1, 3, 2, 1, 2, 1, 2, 1, 3, 2, 1, 2, 1, 3, 2, 1, 3, 2, 1, 2, 1, 2, 3, 1, 2, 3, 1, 2, 1, 2, 3, 1, 2, 1, 2, 1, 2, 1, 3, 2, 3
$31\overline{2}$	1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 3, 2, 1, 3, 1, 2, 1, 2, 3, 1, 2, 3, 1, 3, 2, 1, 3, 2, 1, 3, 1, 2, 3, 1, 2, 1, 2, 3, 2, 1, 2, 1, 3, 1, 3, 2, 1
$3\overline{1}2$	1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 1, 3, 2, 1, 2, 1, 2, 1, 2, 3, 1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 1
$3\overline{12}$	1, 2, 1, 2, 1, 2, 1, 3, 2, 1, 2, 1, 2, 3, 2, 1, 2, 1, 2, 1, 2, 3, 1, 3, 2, 1, 2, 1, 2, 1, 2, 3, 2, 3, 2, 1, 2, 1, 2, 1, 2, 3, 1, 2, 1, 2, 1
$\overline{3}12$	1, 2, 1, 2, 1, 3, 2, 1, 2, 1, 3, 1, 2, 1, 2, 1, 3, 2, 1, 3, 2, 1, 2, 1, 2, 3, 1, 3, 2, 1, 2, 1, 2, 1, 3, 2, 1, 2, 1, 2, 1, 2, 3, 2, 3, 1, 3
$\overline{3}1\overline{2}$	1, 2, 1, 2, 1, 2, 1, 3, 1, 3, 2, 1, 2, 3, 1, 2, 1, 3, 2, 1, 2, 3, 1, 3, 2, 1, 2, 1, 2, 1, 2, 3, 1, 3, 2, 1, 2, 3, 1, 2, 1, 3, 2, 1, 2, 3, 1
$\overline{31}2$	1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 1, 3, 2, 1, 2, 3, 1, 3, 2, 1, 2, 1, 2, 1, 2, 3, 2, 1, 2, 1, 3, 2, 1, 2, 3, 2, 1, 3, 2, 1, 2, 1, 2, 1, 2, 3, 1
$\overline{312}$	1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 1, 2, 1, 2, 1, 3, 1, 3, 2, 1, 2, 1, 2, 3, 1, 2, 3, 1, 2, 1, 2, 1, 3, 1, 2, 1, 2, 3, 1, 2, 1, 2, 1, 2, 1, 3, 2
321	1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 1, 2, 1, 2, 1, 3, 1, 3, 2, 1, 2, 1, 2, 1, 2, 3, 1, 2, 1, 2, 1, 2, 1, 3, 2, 1, 2, 1, 2, 1, 2, 3, 1, 2, 1, 2, 1
$32\overline{1}$	1, 2, 1, 2, 1, 2, 1, 3, 2, 3, 1, 3, 2, 1, 3, 2, 1, 3, 1, 2, 3, 2, 3, 2, 3, 1, 2, 3, 1, 3, 2, 1, 3, 2, 1, 3, 1, 2, 3, 1, 3, 2, 1, 2, 3, 2, 1
$3\overline{2}1$	1, 2, 1, 2, 1, 2, 1, 3, 2, 1, 2, 1, 2, 1, 2, 3, 1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 1, 2, 1, 2, 1, 3, 2, 1, 2, 1, 2, 1, 2, 3, 1, 2, 1, 2, 1, 2, 1
$3\overline{21}$	1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 1, 3, 2, 1, 2, 1, 3, 2, 1, 3, 2, 1, 2, 1, 2, 1, 2, 3, 1, 2, 1, 3, 2, 1, 2, 1, 2, 1, 2, 3, 2, 1, 2, 1, 3, 2, 1
$\overline{3}21$	1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 3, 1, 2, 1, 2, 1, 3, 1, 2, 1, 2, 3, 2, 1, 2, 1, 2, 1, 2, 3, 1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 1, 2, 1, 2, 1, 3, 1
$\overline{3}2\overline{1}$	1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 3, 2, 1, 3, 1, 3, 2, 1, 3, 2, 1, 3, 1, 2, 3, 1, 2, 3, 1, 3, 2, 1, 3, 2, 1, 3, 1, 2, 3, 1, 3, 1, 2, 1, 2, 3, 2
$\overline{32}1$	1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 1, 3, 2, 1, 2, 1, 2, 1, 2, 3, 1, 2, 1, 2, 1, 2, 1, 3, 1, 3, 1, 2, 1, 3, 1, 2, 3, 1, 2, 1, 2, 1, 3, 1, 2, 1, 2
$\overline{321}$	1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 1, 2, 1, 2, 1, 3, 1, 2, 1, 2, 1, 2, 1

It is possible to construct a fault-free Hamiltonian path $HP_{k_1}(\boldsymbol{a}, \boldsymbol{b})$ between two distinct nonfaulty nodes $\boldsymbol{a}$ and $\boldsymbol{b}$ in $BP_{n-1}k_1 - F$ such that the distance between $\boldsymbol{a}$ and $\boldsymbol{b}$ is at most two. Then, $\boldsymbol{a}^{(n)} \in BP_{n-1}k_2$, $\boldsymbol{b}^{(n)} \in BP_{n-1}k_3$, and $k_2 \neq k_3$. From Lemma 1, we can construct a fault-free Hamiltonian path $HP^{\{k_1\}}(\boldsymbol{a}^{(n)}, \boldsymbol{b}^{(n)})$. These paths with edges $(\boldsymbol{a}, \boldsymbol{a}^{(n)})$ and $(\boldsymbol{b}, \boldsymbol{b}^{(n)})$ form a fault-free Hamiltonian cycle as shown in Figure 3.

Case A-2 $|F \cap BP_{n-1}k_1| = n - 3$ and $|F \cap BP_{n-1}k_2| \leq 1$.

In $BP_{n-1}k_1$ and $BP_{n-1}k_2$, construct fault-free Hamiltonian cycles HC_{k_1} and HC_{k_2}, respectively. For $BP_{n-1}k_3$ such that $|k_3| \neq |k_1|, |k_2|$, select nonfaulty nodes $\boldsymbol{a}$ and $\boldsymbol{b}$ in $BP_{n-1}k_1$ and $BP_{n-1}k_2$, respectively, so that $(\boldsymbol{a}, \boldsymbol{a}^{(n)}) \notin F$, $(\boldsymbol{b}, \boldsymbol{b}^{(n)}) \notin F$, $\boldsymbol{a}^{(n)} \in BP_{n-1}k_3$ and $\boldsymbol{b}^{(n)} \in BP_{n-1}k_3$ hold. Moreover, select nodes $\boldsymbol{c}$ and $\boldsymbol{d}$ that are on HC_{k_1} and HC_{k_2}, respectively, and that are neighbor nodes of $\boldsymbol{a}$ and $\boldsymbol{b}$, respectively, so that $(\boldsymbol{c}, \boldsymbol{c}^{(n)}) \notin F$, $(\boldsymbol{d}, \boldsymbol{d}^{(n)}) \notin F$, $\boldsymbol{c}^{(n)} \in BP_{n-1}k_4$, $\boldsymbol{d}^{(n)} \in BP_{n-1}k_5$, and $k_4 \neq k_5$ hold. In $BP_{n-1}k_3$, select a fault-free Hamiltonian path $HP_{k_3}(\boldsymbol{a}^{(n)}, \boldsymbol{b}^{(n)})$ between $\boldsymbol{a}^{(n)}$ and $\boldsymbol{b}^{(n)}$. The path with paths $HC_{k_1} - (\boldsymbol{a}, \boldsymbol{c})$ and $HC_{k_2} - (\boldsymbol{b}, \boldsymbol{d})$, edges $(\boldsymbol{a}, \boldsymbol{a}^{(n)})$, $(\boldsymbol{b}, \boldsymbol{b}^{(n)})$, $(\boldsymbol{c}, \boldsymbol{c}^{(n)})$, and $(\boldsymbol{d}, \boldsymbol{d}^{(n)})$, and path $HP^{\{k_1, k_2, k_3\}}(\boldsymbol{c}^{(n)}, \boldsymbol{d}^{(n)})$ form a fault-free Hamiltonian cycle as shown in Figure 4.

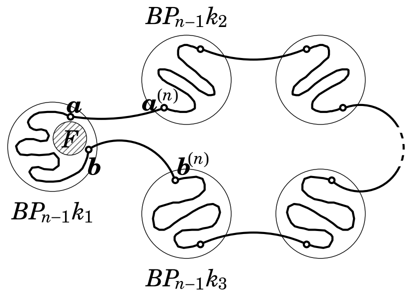

Figure 3: Hamiltonian cycle in case that $F \subset BP_{n-1}k_1$.

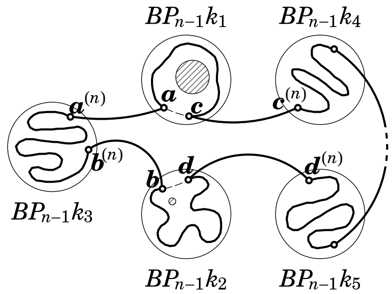

Figure 4: Hamiltonian cycle in case that $|F \cap BP_{n-1}k_1| = n-3$ and $|F \cap BP_{n-1}k_2| = 1$.

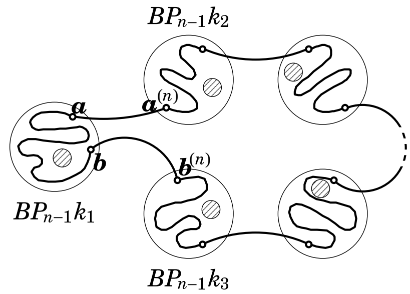

Figure 5: Hamiltonian cycle in case that $\forall k$, $|F \cap BP_{n-1}k| \leq n-4$.

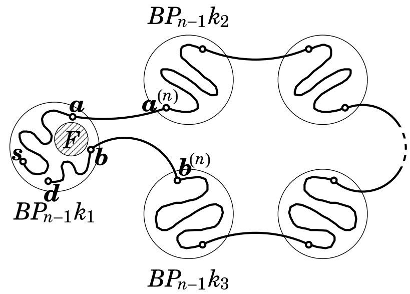

Figure 6: Hamiltonian path in case that $\boldsymbol{s}, \boldsymbol{d} \in BP_{n-1}k_1$, and $F \subset BP_{n-1}k_1$.

Case A-3 $\forall k$, $|F \cap BP_{n-1}k| \leq n-4$.
From the hypothesis of induction, every $BP_{n-1}k$ is Hamiltonian connected. Hence, for some k_1, we can construct fault-free Hamiltonian path $HP_{k_1}(\boldsymbol{a}, \boldsymbol{b})$ between two nonfaulty nodes $\boldsymbol{a}$ and $\boldsymbol{b}$ in $BP_{n-1}k_1$ such that $\boldsymbol{a}^{(n)} \in BP_{n-1}k_2 - F$, $\boldsymbol{b}^{(n)} \in BP_{n-1}k_3 - F$, and $k_2 \neq k_3$ hold. This path and $HP^{\{k_1\}}(\boldsymbol{a}^{(n)}, \boldsymbol{b}^{(n)})$ form a fault-free Hamiltonian cycle as shown in Figure 5.

From above, the former part of the theorem is proved. Next, we show the construction method for a Hamiltonian path in case that $|F| = n-3$.

Case B-1 $\boldsymbol{s}, \boldsymbol{d} \in BP_{n-1}k_1 - F$, and $F \subset BP_{n-1}k_1$.
For $f \in F$, construct fault-free Hamiltonian path $HP_{k_1}(\boldsymbol{s}, \boldsymbol{d})$ between $\boldsymbol{s}$ and $\boldsymbol{d}$ in $BP_{n-1}k_1 - (F - \{f\})$. By deleting f from $HP_{k_1}(\boldsymbol{s}, \boldsymbol{d})$, we obtain two paths between $\boldsymbol{s}$ and $\boldsymbol{a}$ and between $\boldsymbol{b}$ and $\boldsymbol{d}$. Note that the distance between $\boldsymbol{a}$ and $\boldsymbol{b}$ is at most two. Hence, if $\boldsymbol{a}^{(n)} \in BP_{n-1}k_2$ and $\boldsymbol{b}^{(n)} \in BP_{n-1}k_3$ then $k_2 \neq k_3$. The former two paths with edges $(\boldsymbol{a}, \boldsymbol{a}^{(n)})$ and $(\boldsymbol{b}, \boldsymbol{b}^{(n)})$, and path $HP^{\{k_1\}}(\boldsymbol{a}^{(n)}, \boldsymbol{b}^{(n)})$ form a fault-free Hamiltonian path between $\boldsymbol{s}$ and $\boldsymbol{d}$ as shown in Figure 6.

Case B-2 $\boldsymbol{s}, \boldsymbol{d} \in BP_{n-1}k_1$, $F \subset BP_{n-1}k_2$, and $k_1 \neq k_2$.
In $BP_{n-1}k_2$, construct fault-free Hamiltonian cycle HC_{k_2}. Additionally, in $BP_{n-1}k_1$, construct Hamiltonian path $HP_{k_1}(\boldsymbol{s}, \boldsymbol{d})$ between $\boldsymbol{s}$ and $\boldsymbol{d}$. Then, we can select node $\boldsymbol{a}$ on HC_{k_2} and node $\boldsymbol{b}$ on $HP_{k_1}(\boldsymbol{s}, \boldsymbol{d})$ such that $\boldsymbol{a}^{(n)}, \boldsymbol{b}^{(n)} \in BP_{n-1}k_3$. Moreover, we can select $\boldsymbol{c}$ on HC_{k_2} that is adjacent to $\boldsymbol{a}$, and $\boldsymbol{e}$ on $HP_{k_1}(\boldsymbol{s}, \boldsymbol{d})$ that is adjacent to $\boldsymbol{b}$ such that $\boldsymbol{c}^{(n)} \in BP_{n-1}k_4$, $\boldsymbol{e}^{(n)} \in BP_{n-1}k_5$, and k_1, k_4, and k_5 are all distinct. Then, construct fault-free Hamiltonian path $HP_{k_3}(\boldsymbol{a}^{(n)}, \boldsymbol{b}^{(n)})$ between $\boldsymbol{a}^{(n)}$ and $\boldsymbol{b}^{(n)}$ in $BP_{n-1}k_3$. This path with two paths generated by $HP_{k_1}(\boldsymbol{s}, \boldsymbol{d}) - (\boldsymbol{b}, \boldsymbol{e})$, edges $(\boldsymbol{a}, \boldsymbol{a}^{(n)})$, $(\boldsymbol{b}, \boldsymbol{b}^{(n)})$,

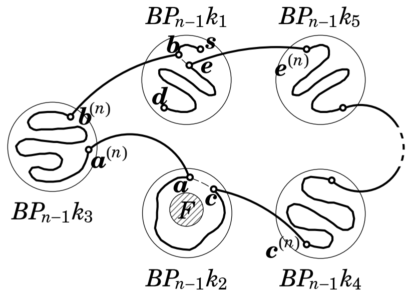

Figure 7: Hamiltonian path in case that $\boldsymbol{s}, \boldsymbol{d} \in BP_{n-1}k_1$, $F \subset BP_{n-1}k_2$, and $k_1 \neq k_2$.

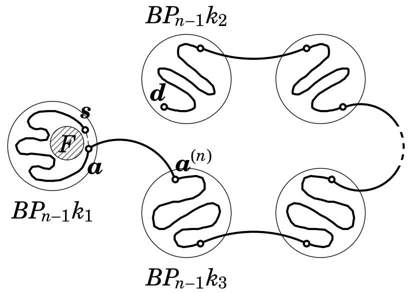

Figure 8: Hamiltonian path in case that $\boldsymbol{s} \in BP_{n-1}k_1$, $\boldsymbol{d} \in BP_{n-1}k_2$, $F \subset BP_{n-1}k$, and $k_1 \neq k_2$, $k \in \{k_1, k_2\}$.

$(\boldsymbol{c}, \boldsymbol{c}^{(n)})$, and $(\boldsymbol{e}, \boldsymbol{e}^{(n)})$, and paths $HC_{k_1} - (\boldsymbol{a}, \boldsymbol{c})$ and $HP^{\{k_1,k_2,k_3\}}(\boldsymbol{c}^{(n)}, \boldsymbol{e}^{(n)})$ form a fault-free Hamiltonian path between $\boldsymbol{s}$ and $\boldsymbol{d}$ as shown in Figure 7.

Case B-3 $\boldsymbol{s} \in BP_{n-1}k_1 - F$, $\boldsymbol{d} \in BP_{n-1}k_2 - F$, $F \subset BP_{n-1}k$, and $k_1 \neq k_2$, $k \in \{k_1, k_2\}$.
We can assume that $k = k_1$ without loss of generality. Construct fault-free Hamiltonian cycle HC_{k_1} in $BP_{n-1}k_1$. Then we can select $\boldsymbol{a}$ on the cycle that is adjacent to $\boldsymbol{s}$ such that $\boldsymbol{a}^{(n)} \in BP_{n-1}k_3$ and $k_3 \neq k_1$. Then, path $HC_{k_1} - (\boldsymbol{s}, \boldsymbol{a})$, edge $(\boldsymbol{a}, \boldsymbol{a}^{(n)})$, and path $HP^{\{k_1\}}(\boldsymbol{a}^{(n)}, \boldsymbol{d})$ form a fault-free Hamiltonian path between $\boldsymbol{s}$ and $\boldsymbol{d}$ as shown in Figure 8.

Case B-4 $\boldsymbol{s} \in BP_{n-1}k_1$, $\boldsymbol{d} \in BP_{n-1}k_2$, $F \subset BP_{n-1}k_3$, and k_1, k_2, and k_3 are all distinct.
In $BP_{n-1}k_3$, construct fault-free Hamiltonian cycle HC_{k_3}. Select $\boldsymbol{a}$ on this cycle so that $\boldsymbol{a}^{(n)} \in BP_{n-1}k_1$ and $\boldsymbol{a}^{(n)} \neq \boldsymbol{s}$ hold. Additionally, select $\boldsymbol{b}$ on HC_{k_3} that is adjacent to $\boldsymbol{a}$ so

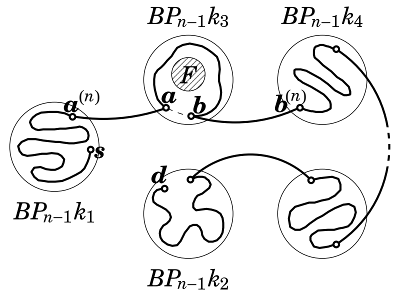

Figure 9: Hamiltonian path in case that $\boldsymbol{s} \in BP_{n-1}k_1$, $\boldsymbol{d} \in BP_{n-1}k_2$, $F \subset BP_{n-1}k_3$, and that k_1, k_2, and k_3 are all distinct.

that $\boldsymbol{b}^{(n)} \in BP_{n-1}k_4$ and $k_4 \neq k_2$ hold. Then, Hamiltonian path $HP_{k_1}(\boldsymbol{s}, \boldsymbol{a}^{(n)})$ in $BP_{n-1}k_1$, edge $(\boldsymbol{a}^{(n)}, \boldsymbol{a})$, path $HC_{k_3} - (\boldsymbol{a}, \boldsymbol{b})$, edge $(\boldsymbol{b}, \boldsymbol{b}^{(n)})$, and path $HP^{\{k_1,k_3\}}(\boldsymbol{b}^{(n)}, \boldsymbol{d})$ form a fault-free Hamiltonian path between $\boldsymbol{s}$ and $\boldsymbol{d}$ as shown in Figure 9.

Case B-5 $\boldsymbol{s} \in BP_{n-1}k_1 - F$, $\boldsymbol{d} \in BP_{n-1}k_2 - F$, $\forall k$, $|F \cap BP_{n-1}k| < n - 3$.
Since every $BP_{n-1}k$ is Hamiltonian connected, we can construct a fault-free Hamiltonian path between $\boldsymbol{s}$ and $\boldsymbol{d}$ from Lemma 1.

Case B-6 $\boldsymbol{s}, \boldsymbol{d} \in BP_{n-1}k_1 - F$, $\forall k$, $|F \cap BP_{n-1}k| < n - 3$.
Since every $BP_{n-1}k$ is Hamiltonian connected, construct fault-free Hamiltonian path $HP_{k_1}(\boldsymbol{s}, \boldsymbol{d})$ between $\boldsymbol{s}$ and $\boldsymbol{d}$ in $BP_{n-1}k_1$. Let $(\boldsymbol{a}, \boldsymbol{b})$ be an edge on $HP_{k_1}(\boldsymbol{s}, \boldsymbol{d})$, and assume that $\boldsymbol{a}^{(n)} \in BP_{n-1}k_2 - F$, $\boldsymbol{b}^{(n)} \in BP_{n-1}k_3 - F$. $HP_{k_1}(\boldsymbol{s}, \boldsymbol{d}) - (\boldsymbol{a}, \boldsymbol{b})$ generates two paths. These paths with edges $(\boldsymbol{a}, \boldsymbol{a}^{(n)})$, $(\boldsymbol{b}, \boldsymbol{b}^{(n)})$, and path $HP^{\{k_1\}}(\boldsymbol{a}^{(n)}, \boldsymbol{b}^{(n)})$ form a fault-free Hamiltonian path between $\boldsymbol{s}$ and $\boldsymbol{d}$ as shown in Figure 10.

From above the latter part of the theorem is also proved. □

5 Conclusion

In this paper, we have proved that a faulty n-burnt pancake graph has a Hamiltonian cycle if the number of the faulty elements is at most $n - 2$, and that it has a Hamiltonian path between any pair of nonfaulty nodes if the num-

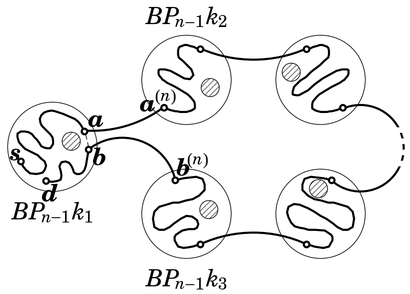

Figure 10: Hamiltonian path in case that $\boldsymbol{s}, \boldsymbol{d} \in BP_{n-1}k_1$, $\forall k$, $|F \cap BP_{n-1}k| < n-3$.

ber of nodes is at most $n-3$. Embedding the Hamiltonian cycles and paths in other Cayley graphs is also attractive.

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